# Coefficient Bounds For QUASI-Subordination Classes

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## Abstract

The bounds for the Fekete-Szego coefficients $|a_3 - \mu a_2^2|$are derived of certain subclasses of star like ,convex and bazilevic functions involving quasi-subordination.

**Key Words** : Analytic functions , univalent function, Bazilevic type function, star like function , Convex function ,Quasi-subordination

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## حدود المعامل لصنوف شبه التابعية

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## الملخص

حدود معاملات فيكيت- سياغو $|a_3 - \mu a_2^2|$ اشتقت لعدد من الصنوف الجزئية للدوال النجمية والمحدبة وداله بازا لفج باستخدام مفهوم شبه التابعية .

**الكلمات المفتاحية** :الدوال التحليلية،الدوال احادية التكافؤ، الدوال النجمية، الدوال المحدبة.

## Introduction

Let $U = \{z : z \in C, |z| < 1\}$ be the open unit disk and A be the class of all analytic functions in $U$ normalized by $f(0) = 0$ and $f'(0) = 1$ of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ .

Any two analytic functions $f$ and $g$ , the function $f$ is subordinate to the function $g$,are written $f \prec g$, provided there is an analytic function $w(z)$ defined on $U$ with $w(0) = 0$ and $| w(z) | < 1$ such that $f(z) = g(w(z))$. In particular,if the function g is univalent in $U$, then f(z) $\prec$ g(z) is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$. see [5] Now, an analytic function $f$ is quasi-subordination to an analytic function $g$ in the open unit disk ,is written f(z) $\prec_q$ g(z) if there exist an analytic function ψ and w , with .$|\psi(z)| \leq 1, w(0) = 0$ and $| w(z) | < 1$ such that $f(z) = \psi(z)g(w(z))$,see[13]

Note that if $\psi(z) = 1$, then $f(z) = g(w(z))$, so that $f(z) \prec g(z)$ in U. Further, if $w(z) = z$, then f(z) = ψ(z)g(z) and it is said that $f$ is majorized to g and written f(z) $<< g(z)$ in $U$.Also, we can see that quasi-subordination is generalization of subordination as well as majorization. For more information see the works related to quasi-subordination in [3,7,10,12].

Let ∅ be an analytic function with positive real part in the unit disk with $U$, $\emptyset(0) = 1$ and $\emptyset(0) > 0$ that maps $U$ onto a region starlike with respect to1 and symmetric with respect to the real axis, Ma and Minda [5] defined the following class

$$S^*(\emptyset) = \left\{ f \in A : \frac{zf'(z)}{f(z)} \prec \emptyset(z) \right\} . \quad (1)$$

A function $f \in S^*(\emptyset)($ is called Ma-Mindastarlike with respect to $\emptyset$.The class $C(\emptyset)$ is the class of function $f \in A$ for which $1 + zf''(z)/f'(z) \prec \emptyset(z)$. The class $S^*(\emptyset)$ and $C(\emptyset)$

include several well-known subclasses of star like and convex functions as special cases. Several authors have discussed various subclasses of the well-known Bazilevic function(see ,for details [4,12]) of type $\lambda$from various viewpoints such as boundary rotational problems ,subordinations relationship, and so on . It is interesting to observe that the earlier investigations on the subject do not seem to have addressed the problems involving coefficient inequalities and coefficient bounds for these subclasses of Bazilevic type functions especially when the parameter λ > 0.

Now, consider the function

$$w(z) = \frac{1 + l(z)}{1 - l(z)} = 1 + w_1 z + w_2 z^2 + w_3 z^3 + \cdots \qquad (2)$$

Then

$$l(z) = \frac{1}{2}(w_1 z + \left(w_{2-} \frac{{w_1}^2}{2}\right) z^2 + \cdots) \qquad (3)$$

Here ,we define the following classes motivated by [9,13,12].

Definition 1.1 :Let the class $S_q^*(\emptyset(z))$consist of functions f satisfying the quasi-subordination

$$\frac{zf'(z)}{f(z)} - 1 \prec_q \emptyset(z) - 1. \qquad (4)$$

**Definition 1.2:**Let the class$R_q$ $(\emptyset(z))$ consist of functions $f$ satisfying the quasi-subordination

$$f'(z) - 1 \prec_q \emptyset(z) - 1. \qquad (5)$$

**Definition 1.3**:Let the class $C_q^*(\emptyset(z))$consist of functions $f$ satisfying the quasi-subordination

$$1+\frac{zf'(z)}{f(z)}-1 \prec_q \emptyset(z)-1. \quad (6)$$

**Definition 1.4**:Let the class $\mathcal{L}_q^*(\emptyset(\alpha,z)),(\alpha \geq 0)$consist of functions $f$satisfying the quasi-subordination

$$\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha}-1 \prec_q \emptyset(z)-1. \quad (7)$$

**Definition 1.5**:Let the class $\mathcal{B}_q^{\nu}(\emptyset(z))$consist of Bazelivic functions $f$ satisfying the quasi-subordination

$$\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{z}\right)^{\lambda}-1 \prec_q \emptyset(z)-1, \quad (8)$$

where λ > 0.

In this paper , we obtain Fekete-Szego inequalities for the classes

$S_q^*(\emptyset(z)), R_q(\emptyset(z), C_q^*(\emptyset(w(z)), \mathcal{L}_q^*(\emptyset(\alpha,z))$, and $\mathcal{B}_q^{\nu}(\emptyset(z))$.Several closely-related function classes are also considered and relevant connections to earlier known results are pointed out.Functions in the above classes are analogous to the α- convex functions of Miller et al.[10] and α-logarithmically convex functions introduced by Lewandowski et al. [8],see also[14].

Many researchers have investigated the bounds of Fekete-Szego coefficient for the various classes [1,2,7,11,12].

In order to prove our results, we need the following lemma due to [6 ]. Lemma 1.6:If $w \in \Omega$ (the class of analytic functions w normalized by $w(0) = 0$ ) and $|w(z)| < 1$,then for any complex number t

$$|w_2 - t w_1^2| \leq \max\{1; |t|\}. \quad (9)$$

The result is sharp for the function $w(z)= z^2$ or $w(z)= z$

## Coefficient bounds

By making use the Lemma 1.6, we prove the following bound for our classes

$S_q^*\big(\emptyset(z)\big), R_q(\emptyset(z), C_q\ (\emptyset(\ w(z)), \mathcal{L}_q^*\big(\emptyset(\alpha, z)\big),$ and$\mathcal{B}_q^{\mathcal{V}}(\emptyset(z))$

Let $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots, \emptyset(z) = 1 + Q_1 + Q_2 z^2 + Q_3 z^3 + \cdots$ and$\psi(z) = c_0 + c_1 z + c_2 z^2$ ,where $Q_1 \in R$ and $Q_1 > 0$.

**Theorem 2.1** : Let $f \in A$ belongs to$S_q^*\big(\emptyset(z)\big)$. Then

$$|a_2| < \frac{Q_1}{2},$$

$$|a_3| \leq \frac{1}{4}(Q_1 + \max\left\{Q_1, \frac{1}{2}(Q_1^2 + |Q_2| - Q_1)\right\})$$

for any complex numberμ

$$\left|a_3 - \mu a_2^2\ \right| \leq \frac{1}{4}(Q_1 + \max\left\{Q_1, \frac{1}{2}(|1-\mu|)Q_1^2 + Q_1 + |Q_2|)\right\})).$$

**Proof**: Let $\in S_q^*\big(\emptyset(z)\big)$.Then there exist analytic functions ψ and $l$, . with$|\psi(z)| \leq 1, l(0) = 0$ and$|l(z) < 1|$such that

$$\frac{zf'(z)}{f(z)} - 1 = \ \psi(z)(\emptyset)l(z)) - 1. \qquad (10)$$

Since

$$\frac{zf'(z)}{f(z)} - 1 = a_2 z + (-a_2^2 + 2a_3)z^2 + \cdots \qquad (11)$$

$$\emptyset\big(l(z)\big) - 1 = \frac{1}{2}Q_1 w_1 z + ( \left|\frac{1}{2}Q_1\left(w_2 - \frac{w_1^2}{2}\right) + \frac{1}{4}Q_2 w_1^2 )z^2 + \cdots \right. \qquad (12)$$

$$\psi(z)(\emptyset)l(z)) - 1 = \frac{1}{2}Q_1 c_0 w_1 z + ( \left|\frac{1}{2}Q_1\left(c_0 w_2 - \frac{c_0 w_1^2}{2}\right) + \frac{1}{4}Q_2 w_1^2 )z^2 + \cdots \right. \qquad (13)$$

It follows that

$$a_2 = \frac{Q_1 c_0 w_1}{2}$$

$$a_3 = \frac{1}{4}[c_0 Q_1 w_1 + Q_2 c_0 w_2 + \frac{1}{2}c_0(Q_1^2 c_0 - Q_1 + Q_2)w_1^2]$$

Since $\psi(z)$ is analytic and bounded in $U$,we have

$$|c_n| < 1 - |c_0|^2 \leq 1 \ (n > 0). \qquad (15)$$

By using (15) and $|w_1| \leq 1$, we obtain

$$a_2 \leq \frac{Q_1}{2}. \qquad (16)$$

Also,

$$a_3 - \mu a_2^2 \ = \ \frac{1}{4}(Q_1 c_1 w_1 + Q_1 c_0 w_2 + \frac{1}{2}c_0(Q_2 - Q_{1+}Q_1^2 c_0 - c_0 Q_1 )w_1^2).$$

Then

$$\left|a_3 - \mu a_2^2\right| \leq \frac{1}{4}(|Q_1 c_1 w_1| + \left|Q_1 c_0 (w_2 - \frac{1}{2}(\frac{Q_2}{Q_1} - 1 + Q_1 c_0 - c_0 )w_1^2)\right|).$$

By using $|c_n| \leq 1$ and $|w_1| \leq 1$, we have

$$\left|a_3 - \mu a_2^2\right| \leq \frac{Q_1}{4}(1 + \left|w_2 - \frac{1}{2}(-(1-2\mu)Q_1 c_0 + 1 - \frac{Q_2}{Q_1})w_1^2)\right|.$$

Applying Lemma 1.6 to

$$\left| w_2 - \frac{1}{2}\left(-(1-2\mu)Q_1\ c_0 + 1 - \frac{Q_2}{Q_1}\right) w_1^2 \right|,$$

Yields

$$\left|a_3 - \mu a_2^2\right| \le \frac{Q_1}{4}(1 \quad + max 1, \left|\frac{1}{2}\left(-(1-2\mu)Q_1\ c_0 + 1 - \frac{Q_2}{Q_1}\right)\right|.$$

Note that

$$\frac{1}{2}\left(-(1-2\mu)Q_1\ c_0 + 1 - \frac{Q_2}{Q_1}\right) \le \frac{1}{2}\left(Q_1\ c_0|1-2\mu| + 1 + \left|\frac{Q_2}{Q_1}\right|\right),$$

therefore, we have

$$\left|a_3 - \mu a_2^2\right| \le \frac{Q_1}{4}(1 \quad + max \quad \{1, \frac{1}{2}\left(|(1-2\mu)|Q_1 \quad + 1 + \left|\frac{Q_2}{Q_1}\right|\right\})).$$

For μ = 0, the above will reduce to $|a_3|$.

**Theorem 2.2** : Let $f \in A$ and satisfies

$$\frac{zf'(z)}{f(z)} - 1 \ll \emptyset(z) - 1.$$

Then , we have

$$|a_2| \le \frac{Q_1}{2}$$

$$|a_3| \le \frac{1}{4}\left(Q_1 + \frac{1}{2}\left(Q_1^2 + |Q_2| - Q_1\right)\right)$$

and, for any complex number μ, $\left|a_3 - \mu a_2^2\right| \le \frac{1}{4}\left(Q_1 + \frac{1}{2}\left(|1-\mu|\right)Q_1^2 + Q_1 + |Q_2|\right)$

**Proof**:The required result obtains by setting $l(z) = z$ in the proof of previous theorem

**Theorem 2.3** : Let $f \in A$ belongs to$R_q^*(\emptyset(z)$ .Then

$$|a_2| \le \frac{Q_1}{4}$$

$$|a_3| \le \frac{1}{6}(Q_1 + \max\{Q_1, \frac{1}{2}(Q_1 + |Q_2|)\})$$

for any complex number μ

$$|a_3 - \mu a_2^2| \le \frac{1}{6}Q_1 + \max\{Q_1, \frac{1}{2}\left(Q_1 + \frac{3}{4}Q_1^2|\mu| + |Q_2|\right)\})$$

**Proof**: Let $f \in R_q^*(\emptyset(z))$.Then there exist analytic functions $\psi$ and , with $|\psi(z)| \le 1$

$l(0) = 0$ and $|l(z) < 1|$ such that

$$f'(z) - 1 = \psi(z)(\emptyset)l(z)) - 1. \tag{17}$$

Since

$$f'(z) - 1 = 2a_2z + 3a_3z + \cdots \tag{18}$$

It follows that from(17),(18)

$$a_2 = \frac{Q_1c_0w_1}{4} \tag{19}$$

$$a_3 = \frac{1}{6}[c_1Q_1w_1 + Q_1c_0w_2 + \frac{1}{2}c_0(Q_1 - Q_2)w_1^2].$$

Since $\psi(z)$ is analytic and bounded in $U$,we have

$$|c_n| < 1 - |c_0|^2 \leq 1 \ (n > 0). \qquad (20)$$

By using (20) and $|w_1| \leq 1$, we obtain

$$a_2 \leq \frac{Q_1}{4}. \qquad (21)$$

Also, by using $|c_n| \leq 1$ and $|w_1| \leq 1$, we have

$$|a_3 - \mu a_2^2| \leq \frac{Q_1}{6}\left(1 + \left|w_2 - \frac{1}{2}(1 + \frac{3}{4}c_0\mu Q_1 - \frac{Q_2}{Q_1})w_1^2\right|\right).$$

Applying Lemma 1.6 to

$$\left|w_2 - \frac{1}{2}(1 + \frac{3}{4}c_0\mu Q_1 - \frac{Q_2}{Q_1})w_1^2\right|,$$

yields

$$|a_3 - \mu a_2^2| \leq \frac{Q_1}{6}\left(1 + max1, \frac{1}{2}\left|1 + \frac{3}{4}c_0\mu Q_1 - \frac{Q_2}{Q_1}\right|\right).$$

Since

$$\frac{1}{2}\left|1 + \frac{3}{4}\mu c_0 Q_1 - \frac{Q_2}{Q_1}\right| \leq \frac{1}{2}\left(1 + \frac{3}{4}|\mu||c_0|\,Q_1 - \left|\frac{Q_2}{Q_1}\right|\right),$$

and $|c_0| \leq 1$ therefore, we have the result

**Theorem 2.4 :** Let$f \in A$ and satisfies

$$f'(z) - 1 \ll \emptyset(z) - 1.$$

Then , we have

$$|a_2| \leq \frac{Q_1}{4}.$$

$$|a_3| \leq \frac{1}{6}(Q_1 + \frac{1}{2}(Q_1 + |Q_2|)).$$

and, for any complex number μ,

$$|a_3 - \mu a_2^2 \ | \le \frac{1}{6}(Q_1 \quad + \frac{1}{2}(Q_1 \ + \frac{3}{4}Q_1^2|\mu| + |Q_2|)).$$

**Proof**:The required result obtains by setting $l(z) = z$ in the proof of previous theorem

**Theorem 2.5** : Let $f \in A$ belongs to$C_q^*(\emptyset(z))$.Then

$$|a_2| \le \frac{Q_1}{4}.$$

$$|a_3| \le \frac{1}{12}(Q_1 + \max\{Q_1, \frac{1}{2}\left(1 + Q_1^2 + \left|\frac{Q_2}{Q_1}\right|\right)\})$$

for any complex number μ

$$|a_3 - \mu a_2^2 \ | \le \frac{1}{12}\left(Q_1 + \max\left\{Q_1, \frac{1}{2}\left(1 + Q_1^2\left|1 - \frac{3}{2}\mu\right| + \left|\frac{Q_2}{Q_1}\right|\right)\right\}\right).$$

**Proof**: Let $f \in C_q^*(\emptyset(z))$. It is easy to show that

$$\left(1 + \frac{zf'(z)}{f(z)} - 1\right) - 1 = 2a_2 z \quad + 2\left(-2a_2^2 \quad + 3a_3\right) + z + \cdots.$$

The proof can now be completed as in the proof of Theorem 2.1.

**Theorem 2.6 :** Let$f \in A$ and satisfies

$$\frac{zf'(z)}{f(z)} - 1 \ll \emptyset(z) - 1.$$

Then , we have

$$|a_2| \le \frac{Q_1}{4}.$$

$$|a_3| \le \frac{1}{12}(Q_1 + \frac{1}{2}(1 + Q_1^2 + |Q_2|))$$

and, for any complex number μ, $|a_3 - \mu a_2^2 \;| \le \frac{1}{12}(Q_1 + \frac{1}{2}\left(Q_1 + Q_1^2\left|1 - \frac{3}{2}\mu\right|) + |Q_2|\right)$

**Proof:** The required result obtains by setting $w(z) = z$ in the proof of previous theorem

**Theorem 2.7** Let $f \in A$ belong to $\mathcal{L}_q^*(\emptyset(z)$ , $\alpha \ge 0$ *and* $\beta = 1 - \alpha$. Then

$$|a_2| \le \frac{Q_1}{2|\alpha + 2\beta|},$$

$$|a_3| \le \frac{Q_1}{4|\alpha + 2\beta|}\ (2Q_1 + \max\left\{2Q_1, Q_1 + \frac{|2((\alpha+2\beta)^2 - 3(\alpha+4\beta))|}{(\alpha+2\beta)^2}Q_1^2 + |Q_2|\right)\}),$$

for any complex number μ

$$|a_3 - \mu a_2^2 \;| \le \frac{Q_1}{4|\alpha + 2\beta|}\ (2Q_1 + \max\left\{2Q_1, Q_1 + \frac{|2((\alpha+2\beta)^2 - 3(\alpha+4\beta)) - \mu(\alpha+3\beta)|}{(\alpha+2\beta)^2}Q_1^2 + |Q_2|\right)\}),$$

**Proof**: If $f \in \mathcal{L}_q^*(\emptyset(z)$ .It is easy to show that

$$\frac{zf'(z)^{\alpha}}{f(z)}\left(1 + \frac{zf''(z)}{f'(z)}\right)^{\beta} - 1 = (\alpha + 2\beta)a_2 z \quad + \frac{1}{2}(((\alpha+2\beta)^2 - 3(\alpha+4\beta){a_2}^2 + 4(\alpha+3\beta)a_3)z^2 + \cdots$$

$$(\emptyset)l(z) - 1 = \frac{1}{2}\ Q_1 w_1 z + (\left|\frac{1}{2}\ Q_1\left(w_2 - \frac{{w_1}^2}{2}\right) + \frac{1}{4}Q_2{w_1}^2)z^2 + \cdots \quad (22)$$

$$\psi(z)(\emptyset)l(z) - 1 = \frac{1}{2}\ Q_1 c_0 w_1 z + (\left|\frac{1}{2}\ Q_1\left(c_0 w_2 - \frac{c_0{w_1}^2}{2}\right) + \frac{1}{4}Q_2{w_1}^2)z^2 + \cdots \quad (23)$$

The proof is similar to the proof of Theorem 2.1.

**Theorem 2.8 :** Let $f \in A$ and satisfies

$$\frac{zf'(z)^{\alpha}}{f(z)}\left(1+\frac{zf''(z)}{f'(z)}\right)^{\beta}-1 \ll \emptyset(z)-1.$$

Then , we have

$$|a_2| \leq \frac{Q_1}{2|\alpha+2\beta|},$$

$$|a_3| \leq \frac{Q_1}{4|\alpha+3\beta|}\ (3Q_1+\frac{\left|2\left((\alpha+2\beta)^2-3(\alpha+4\beta)\right)\right|}{(\alpha+2\beta)^2}Q_1^2+|Q_2|),$$

And for any complex number μ

$$\left|a_3-\mu a_2^2\ \ \right| \leq \frac{Q_1}{4|\alpha+3\beta|}\ (3Q_1+\frac{\left|2\left((\alpha+2\beta)^2-3(\alpha+4\beta)\right)-\mu(\alpha+3\beta)\right|}{(\alpha+2\beta)^2}Q_1^2+|Q_2|)$$

**Proof**:The required result obtains by setting $l(z) = z$ in the proof of previous theorem

**Theorem 2.9**:Let $f \in A$ belong to$\mathcal{B}_q^{\gamma}(\emptyset(z)$. Then

$$|a_2| \leq \frac{Q_1}{\lambda+1},$$

$$|a_3| \leq \frac{1}{\lambda+1}\ (\ Q_1+\max\left\{Q_1, Q_1^2\ +\left|\frac{(\lambda+2)(\lambda-1)}{2(\lambda+1)^2}\right|+\ \ |Q_2|\right\})),$$

for any complex number μ

$$\left|a_3-\mu a_2^2\ \ \right| \leq \frac{1}{\lambda+1}\ (\ Q_1+\max\left\{Q_1, Q_1^2\ +\left|\frac{(\lambda+2)(\lambda-1)-\mu}{2(\lambda+1)^2}\right|+\ \ \left|\frac{Q_2}{Q_1}\right|\right\})),$$

**Proof**: Let $f \in \mathcal{B}_q^{\gamma}(\emptyset(z))$. Then there exist analytic functions $\psi$ and , with$|\ \psi(z)| \leq 1, l(0)=0$ and$|l(z)<1|$such that

$$\frac{zf'(z)}{f(z)}(\frac{f(z)}{z})^{\lambda}-1=\ \psi(z)(\emptyset)l(z)-1. \qquad (24)$$

Since

$$\frac{zf'(z)}{f(z)}(\frac{f(z)}{z})^{\lambda}-1=a_2(\lambda+1)z+\frac{1}{2}(\lambda+2)(a_2^2(\lambda-1)+2a_3))z^2+\cdots \quad (25)$$

$$\phi\big(w(z)\big)=\ Q_1w_1z+(\ Q_1w_2+Q_2{w_1}^2)z^2+\cdots \quad (26)$$

$$\psi(z)(\emptyset)l(z)-1)=\ Q_1c_0w_1z+(\ Q_1c_1w_1-c_0(\ Q_1w_2+Q_2{w_1}^2)z^2+\cdots \quad (27)$$

It follows that

$$a_2\leq\frac{Q_1c_0w_1}{\lambda+1}, \quad (28)$$

$$a_3\leq\frac{1}{\lambda+1}\ [\ Q_1c_1w_1+\ Q_1c_0w_2+c_0(\ Q_2-\frac{Q_1^2c_0(\lambda-1)(\lambda+2)}{2(\lambda+1)^2}{w_1}^2]\,. \quad (29)$$

Since $\psi(z)$is analytic and bounded in $U$,we have

$$|c_n|<1-|c_0|^2\leq 1\ (n>0). \quad (30)$$

By using (30) and $|w_1\ |\leq 1$, we obtain

$$a_2\leq\frac{Q_1}{\lambda+1} \quad (31)$$

Also ,

$$a_3-\mu a_2^2=\frac{1}{\lambda+1}\ (Q_1c_1w_1+c_0(Q_1w_2+(\ Q_2-\frac{Q_1^2c_0(\lambda-1)(\lambda+2)-2\mu}{2(\lambda+1)^2}Q_1^2c_0)w_1^2)\,.$$

Then

$$|a_3-\mu a_2^2|\leq\frac{1}{\lambda+1}|Q_1c_1w_1|+|Q_1c_0(w_2+(Q_2 -\frac{Q_1^2c_0(\lambda-1)(\lambda+2)-2\mu}{2(\lambda+1)^2}Q_1^2c_0)w_1^2\quad ).$$

By using $|c_n| \leq 1$ and $|w_1| \leq 1$, we have

$$|a_3 - \mu a_2^2| \leq \frac{Q_1}{\lambda+1}\left(1 + \left| w_2 - \frac{(\lambda+2)(\lambda-1)-2\mu}{2(\lambda+1)^2} Q_1 c_0 - \frac{Q_2}{Q_1}) w_1^2 \right|\right.$$

Applying Lemma 1.6 to

$$\left| w_2 - \frac{(\lambda+2)(\lambda-1)-2\mu}{2(\lambda+1)^2} Q_1 c_0 - \frac{Q_2}{Q_1}) w_1^2 \right|,$$

yields

$$|a_3 - \mu a_2^2| \leq \frac{Q_1}{\lambda+1}\left(1 + max1 \left| \frac{(\lambda+2)(\lambda-1)-2\mu}{2(\lambda+1)^2} Q_1 c_0 - \frac{Q_2}{Q_1}) \right|\right..$$

Note that

$$\left| \frac{(\lambda+2)(\lambda-1)-2\mu}{2(\lambda+1)^2} Q_1 c_0 - \frac{Q_2}{Q_1}) w_1^2 \right| \leq Q_1 |c_0| \left| \frac{(\lambda+2)(\lambda-1)-2\mu}{2(\lambda+1)^2} + \left|\frac{Q_2}{Q_1}\right|) \right|,$$

therefore, we have

$$|a_3 - \mu a_2^2| \leq \frac{Q_1}{\lambda+1}\left(1 + \max\{1, Q_1\} \left| \frac{(\lambda+2)(\lambda-1)-2\mu}{2(\lambda+1)^2} + \left|\frac{Q_2}{Q_1}\right|) \right|\right..$$

For $\mu = 0$, the above will reduce to $|a_3|$.

**Theorem 2.10 :** Let $f \in A$ and satisfies

$$\frac{zf'(z)}{f(z)}(\frac{f(z)}{z})^{\lambda} - 1 \ll \emptyset(z) - 1.$$

Then , we have

$$|a_2| \leq \frac{Q_1}{\lambda + 1}$$

$$|a_3| \leq \frac{1}{\lambda + 1}\left( Q_1 + |\, Q_2| + \frac{Q_1^2(\lambda - 1)}{2(\lambda + 1)}\right).$$

and, for any complex number μ,

$$|a_3 - \mu a_2^2| \leq \frac{1}{\lambda + 1}\ (\ Q_1 + \left|\frac{(\lambda + 2)(\lambda - 1) - 2\mu}{2(\lambda + 1)^2}\right| Q_1^2 + |\, Q_2|\})))\ .$$

**Proof**:The required result obtains by setting $l(z) = z$ in the proof of previous theorem.

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