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Determinations of Compression Buckling for Long and Intermediate Aluminum Alloy 2014-T4 Columns

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ARTICLE INFO	ABSTRACT
Article history: Received January 18, 2022 Accepted March 5, 2022	Experimental tests were carried out on long and intermediate columns of 2014-T4 aluminum alloy stressed to compression buckling. 24 samples were used, divided into two groups (12 specimens before shot peening and ultrasonic impact peening (SP +UIP) and the rest ofter (SP +UIP).
<i>Keywords:</i> 2014-T4 aluminum alloy Shot Penning (SP) Ultrasonic Impact Treatment (UIT) Critical buckling load (Pcr)	• and the rest after (SP +UIP)). The ultimate tensile strength (UTS) and yield strength (YS) were improved due (SP +UIP) by 2.60% and 3.030%, respectively. ANSYS 18.2 (APDL) with the theories of Euler and Johnson was theories was used to estimate the critical buckling load (Pcr) and compare the results with the experimental work. It was found that there was strong agreement between the theories of Euler and Johnson and theories and the experimental work, with safety factors of (1.6, 2.3) and (1.7, 2.5) for long and medium columns before and after the experiment, respectively (SP +UIP). In contrast, ANSYS 18.2 (APDL) provided good predictions with a safety factor of (2.1 and 2.6) and (1.8 and 2.6) for long and medium columns before and after (SP +UIP), respectively.

1. Introduction

Buckling analysis is critical for axially loaded parts because the applied compressive stress at the moment of failure is less than the ultimate compressive stress of the material. In the design of axially compressed parts components, the compressive load and the component shape of the part must be carefully considered to ensure that failure does not occur due to elastic instability and to improve part performance such as fatigue, wear and friction. Among these advances in surface enhancement, shot penning (SP) is a popular and commonly used process that has long been employed in aircraft segments to maximise component efficiency. Ultrasonic impact treatment (UIT) is a newly developed technique investigated in this report. Due to its precise position and reliable operation [1]. Despite being more expensive

than shot peening due to its lower power volume, shot peening and ultrasonic peening may be used for a wide range variety of aircraft parts, including fuselage, cutting edges and with excellent repeatability wings. and durability, according to the manufacturer. Shot peening and ultrasonic peening create deep compressive residual stresses in the surface layer that delay or prevent fractures, extend fatigue blockage at this stage, and limit service life [2]. For steel structures, stability is a critical limit state, both during construction and during service life. The complexity of this process and the different material properties influenced by geometry, material defects and non-linearity of the material make the calculation of the critical load for structural stability a challenge [3]. When a component suddenly fails due to high compressive stress, it is called buckling. When

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an axially loaded part fails, buckling analysis is required because the applied compressive stress is less than the ultimate compressive stress of the material. Therefore, when designing axially special consideration loaded components, should be given to the component geometry and compressive loading to ensure that failure is not caused by elastic instability [4]. Engineers have traditionally paid great attention to calculating the structural stability of their structures. In particular, since Euler determined the critical buckling load for a simply supported column in 1744, estimating the critical buckling load of a structure has been a focus of study. Buckling occurs when a structure abruptly transitions from one state of equilibrium to another. Assessing the buckling loads of a structure is crucial because there is a risk of rapid failure when the critical buckling load is reached [5]. Certain components can lose their stability when subjected to buckling loads. Aluminum alloys are widely used for construction applications such as buildings, bridges and other special structures due to their numerous advantages as lightweight, improved corrosion such resistance, high strength-to-weight ratio and manufacturing flexible process through extrusion. For these reasons, aluminium alloy (AA) has gained popularity in construction [6]. AL -Khazraji A. N. et al studied the effects of combined axial and horizontal lateral side load on the buckling behavior of the I-beam. Using the theory of elastic statics, a theoretical formulation was established to determine the critical buckling load for this combined loading configuration.[7]

Saad T. Faris .et al, discovered the combination Shot Penning (SP) and Ultrasonic Impact Treatment (UIT) to improve the buckling strength of AA 2014-T4 Perry -Robertson and ANSYS were used to determine the critical buckling load and compare it with the experimental result which showed good agreement with the theoretical estimate. [8] Saad T. Faris et al, Cumulative Fatigue damage of AA7075 - T6 under Shot Peening and Ultrasonic Surface Treatments. It was found that the theories of Euler and Jonson compared with the experimental results gave agreement with a factor of safety of 1.8 and 2.4 and 1.8 and 2.4 for long and intermediate columns before and after (SP-UIP), respectively.[9]

Al-Alkawi et al investigated the effect of 10 min wet and dry shot peening on the mechanical properties of 7075-T6 an alloy. The mechanical properties (tensile strength and yield strength) were increased by (2.42%, 4.1% and 3.23%, 5.66%) by dry and wet shot peening, respectively. [10]

Surface treatments have been found in the literature to have a significant impact on the durability and strength of buildings. The current study focuses on the effects of combined surface treatments (shot penning (SP) and ultrasonic impact penning (UIP)) on the column surface using three types of theories (Euler, Johnson and ANSYS) and compares the results findings with experimental.

Ali et al. studied the impacts of shot peening duration on the mechanical characteristics of aluminum alloys 2017-T4 and 6063-T5. The test was carried out using conventional tensile specimens and shot peening times varying. The yield and tensile strength values were found to be higher. The best improvement owing to shot peening was reported for AA 2017-T4 and AA 6063-T5 at (15 and 9) minutes, respectively.[11]

Yusuf et all on 7075 aluminum alloy, researchers investigated the effects of pure severe shot peening (SSP), pure ultrasonic nanocrystalline surface modification (UNSM), and the combination of these two treatments. Detailed microstructure analysis. surface roughness, micro-hardness, and wear-friction testing were performed on the treated samples. The UNSM + SSP and SSP + UNSM treatments significantly enhanced the hardness of the surface layer. Two-stage operations of UNSM + SSP and SSP + UNSM improved friction and wear performance significantly. The results have also been consistent with residual stress, hardness, and nano crystallization studies [12].

2. The work's objectives

The objectives of this work are as follows:

- 1. investigate the buckling behavior of the column in the fixed-pinned state.
- 2. Examining the influence of surface treatments (SP and UIT) on the buckling behaviour of AA 2014-T4 specimens.

- 3. Theoretically, estimating the critical load of the columns using (Euler, Johnson) calculations.
- 4. Calculate the buckling load numerically using ANSYS 18.2 (APDL).
- 5. A comparison of the experimental data produced using the Euler, Johnson, and ANSYS formulae with a statement of which formula is better acceptable with the practical results.3. Experiments

3.1 Composition of chemicals

The analysis of the chemical composition of the material used in this work was carried out with an optical emission spectrometer, which allows computerized processing at the State for Engineering Rehabilitation and Inspection (SIER). The results were compared with the American standard as shown in Table (1).

2014-T4 Aluminium alloy	Al	Cr	Cu	Fe	Mg	Mn	Si	Ti	Zn
ASM [13]	Balance	Max 0.1	3.9 - 5	Max 0.7	0.2 - 0.8	0.4 - 1.2	0.5 - 1.2	Max 0.15	Max 0.25
Experimental	Balance	0.08	4.1	0.36	0.52	0.96	0.8	0.11	0.19

Table 1: Chemical composition of 2014-T4 Aluminium alloy

3.2 Tensile test

By ASTM requirements, the mechanical characteristics of the (2014-T4 Aluminium

alloy) were measured and characterized [13,14]. Figure (1) shows the tensile test specimen (all dimensions in mm).



Figure 1. Size and shape of tensile specimen in accordance with ASTM standards [14]

The tensile test was carried out at the University of Technology-Material Engineering Department with the help of a WDW-200E device with a capacity of 200KN. The

experimental findings represent the mean of three different specimens. The mechanical properties were given in table (2), and plotted in figure (2):

2014 T4 —			
2014-14 Aluminium alloy	UTS (MPa)	YS (MPa)	E (GPa)
Standard ASM [13]	425	290	73
As received	407	281	70
		Specimen after (SP+UIT)	
As received	420	291	72

Table 2: The mechanical properties of 2014-T4 Aluminium alloy



Figure 2. Stress-strain curve of 2014-T4 Al alloy before and after (SP+UIT)

Figure (2) shows that after treating the surface of the samples with (SP+UIT), the ultimate tensile strength (UTS) and yield strength (YS) increased from (410 and 284) to (422 and 293) respectively, and the improvement percentage was (2.84 and 3.07 %) for the ultimate tensile strength (UTS) and the

yield strength (YS) before and after (SP+UIP) respectively.

3.3 Buckling specimens' dimensions

The dimensions of the 2014-T4 Aluminium alloy specimens utilized in this experiment are listed in Table (3).

Table 3: Buckling specimens' dimensions (Aluminium alloy 2014-T4)

NO.	L _T (mm)	L _e (mm)	D (mm)	S.R	Cc	Type of column
1	500	350	10	140	70.21	long
2	500	350	8	175	70.85	long
3	500	350	6	233.333	71.11	long
4	400	280	10	112	70.21	long
5	400	280	8	140	70.85	long
6	400	280	6	186.66	71.11	long
7	300	210	10	84	70.21	long
8	300	210	8	105	70.85	long
9	300	210	6	140	71.11	long
10	200	140	10	56	70.21	intermedia te
11	200	140	8	70	70.85	intermedia te
12	200	140	6	93.33	71.11	long

S.R: Slenderness Ratio, C_c: Column Constant S.R = L_e / r_{min}

 $C_{c} = \sqrt{\frac{2\pi^{2}E}{\sigma_{y}}}$

Where E: is the modulus of elasticity of the column material, σ_y : is yield stress of column material (in tension).

The column constant is clearly dependent on the mechanical characteristics of the material employed.

Short columns, large columns, & the three kinds of columns are columns of moderate length. Table1 shows the three types of columns for various materials based on the slenderness ratio (S.R.). [15]

	Long column	Intermediate column	Short column
Material	(Elastic stability limit)	(Inelastic stability limit)	(strength limit)
Structural steel	SR>150	40< SR< 150	SR<40
Aluminium Alloy	SR> 75	12< SR<75	SR<12
Wood	(18-30) <sr< 50<="" td=""><td>11 < SR < (18-30)</td><td>SR<11</td></sr<>	11 < SR < (18-30)	SR<11

Table 4: Slenderness ratio of columns for different	t materials [16]
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3.4 Buckling test

In this study, the 2014-T4 aluminium alloy specimens were tested by using the rotating buckling device, which is capable of buckling the columns by applying an axial compression load at two different rotating speeds (17 and 34 rpm). The details of the test rig and fixed buckling specimens are depicted in Figure (3). The test had done at the University of Technology, Electromechanical Engineering Department.



Figure 3. Rotating buckling device

3.5 Surface treatment

The mechanical characteristics and the resistance to the buckling load were improved by using ultrasonic impact treatment (UIT) and shot peening (SP) together.

3.5.1 Ultrasonic Impact Peening Treatment (UIP)

By employing an ultrasonic impact peening (UIP) device, the mechanical properties of the 2014-T4 aluminium alloy were enhanced. The UIP device is composed of two parts (a generator handheld component and a component), with the handheld component displayed in figure (4) and its specifications mentioned in table (5). The UIP extracts the ultrasonic energy and inserts it into the metals by surface impulse contact with the metals. A harmonic / resonant of an acoustically tuned body is converted to mechanical impulses on a surface, and this energy is transferred into the metal during the process [17].



Figure 4. UIT device's handheld component

 Table 5: The UIT Machine for specifications [15]

Items	Values
Major power supply	220 V, 50HZ
Common max. working current	4.5 A
DC fuse wire diameter	4.55 A
Max. pulse power	1000 W
Matched transducer	20 KHz
Recommend max. power working	500 W
Impact needle	4 sets, Ø 3×25 mm

3.5.2 Shot Peening (SP)

Shot peening can be carried out with the help of a spinning wheel system. The wheel has a diameter of 590 mm and rotates at a speed of 1435 revolutions per minute. The flow rate of the abrasive is varied to achieve different degrees of consolidation. The shot peening machine used in this study was designated as (STB-OB) and the following specifications of the machine are given in Table (6).

Items	Quant.	Unit	Remark
Ball size	0.6	mm	
Sphere material			Cast Steel
Rockwell hardness	(48 - 50)	HRC	
Pressure	12	bar	
Speed	40	m/sec	
Distance from nozzle	10		
to specimen	10	cm	

(2)

 Table 6: Machine for shot penning specifications

4. Buckling theories

Based on the slenderness ratio (SR) and the column constant, the columns are categorized into two groups: long and intermediate (Cc). If (SR) is more than (Cc), the column is considered long and the Euler formula should be used to investigate the column length. If (SR) is less than (Cc), it is intermediate, and should be calculated using the J.B. Johnson formula [16].

Slenderness Ratio (SR): is the ratio of the effective length of column to its least radius of gyration, as shown in figure (5):

 $S.R = KL / r_{min} = L_e / r_{min}$ (1) where K: end-fixity factor

$$r_{\min} = \sqrt{\frac{I}{A}}$$



Figure 5. Effective length (L_e) for fixed-pinned condition

Column Constant (
$$C_c$$
)

$$C_{c} = \sqrt{\frac{2\pi^{2}E}{\sigma_{y}}}$$
(3)

Where σ_{v} : yield stress of column material

4.1 Euler formula

The Euler formula is defined as the proof under the following assumptions: perfect straightness of the column and true axiality of the applied load; the uniform cross-sectional shape of the column along its length; perfectly elastic, isotropic and homogeneous material; large column length in relation to the crosssectional dimensions; and neglect of direct pressure reduction [6]. In the present work, the column is fixed at one end and pinned at the other, as shown in Figure (6) [19].



Figure 6. Column with one end pinned, the other one fixed

$$P_{\rm cr E} = \frac{\pi^2 E I}{(KL)^2} \tag{4}$$

where P_{cr E}: Critical load (Euler)

4.2 J.B. Johnson formula

The equation below is one form of a group of equations that called parabola formulas [18]:

$$P_{crj} = A\sigma_{y} \left[1 - \frac{\sigma_{y} (\frac{L_{e}}{r})^{2}}{4\pi^{2}E} \right]$$
(5)

 $P_{cr j}$: Critical load (Johnson) A: cross section area.

3.3 ANSYS model

The FE analysis in this study was carried out using ANSYS 18.2 (APDL). The critical buckling load of 2014-T4 aluminium alloy columns is calculated using ANSYS. Thus, it is able to predict the theoretical buckling strength of an ideal elastic structure. It calculates the structural eigenvalues considering the system loads and limits. As shown in figures (7)(8):



Figure 7. The deflection shape of lateral buckling for long column



Figure 8. The deflection shape of lateral buckling for intermediate column

5. Results and discussions

5.1 Buckling results

A comparative analysis of the critical buckling load (P_{cr}) of the dry columns before

(SP +UIT) is shown in tables (7) and (8) for the long and medium columns with and without safety factors, respectively, using different models (Experimental, Euler, Johnson and ANSY.

Table 7: Comparison of (Euler and ANSYS) findings with experimental work for long columns prior to (SP+UIT)

Sp.	L	Leff	D	P _{cr} Exper.	Pcr Euler	$P_{\rm cr} {\rm Euler} {\rm S.} {\rm F} = 1.8$	Per ANSYS	P_{cr} ANSYS S. F=
NO.	(mm)	(mm)	(m)	(N)	(N)	(N)	(N)	2.2 (N)
1	500	350	10	1560	2727.6	1516	2801	1271
2	500	350	8	813.8	1121.4	612.2	1125	511.4
3	500	350	6	216.1	350.09	190.8	353.8	155.4
4	400	280	10	2827	4283.2	2365	4367	1980
5	400	280	8	1055	1738.5	973.9	1795	814.8
6	400	280	6	360.4	546.39	306.1	566.3	257.3
7	300	210	10	4609	7622.3	4210	7756	3510
8	300	210	8	2131	3115.2	1727	3166	1418
9	300	210	6	603.7	966.14	546.5	1006	451.9
10	200	140	6	1435.2	2213.6	1216	2257	1020

 Table 8: Comparison of (Johnson and ANSYS) findings with experimental work for intermediate columns prior to (SP+UIT)

Sp. No.	L (mm)	Leff (mm)	D (mm)	P _{cr} Exper. (N)	P _{cr} Johnson (N)	P _{cr} Johnson S. F= 2.5 (N)	P _{cr} ANSYS (N)	P _{cr} ANSYS S. F= 2.7 (N)
1	200	140	10	6663	15213	6061.2	17351	6423.560
2	200	140	8	3155	7167	2850.9	7122	2625.21

Tables (9) and (10) illustrate a comparison between the three methods with the

experimental work after (SP+UIT) for long and intermediate columns respectively.

 Table 9: Comparison of (Euler and ANSYS 18.2 (APDL)) results with experimental results for long columns following (SP+UIT)

Sp. No.	L (mm)	L _{eff} (mm)	D (mm)	P _{cr} Exper. (N)	P _{cr} Euler (N)	P _{cr} Euler S. F= 1.8 (N)	P _{cr} ANSYS (N)	P _{cr} ANSYS S. F= 1.9 (N)
1	500	350	10	1707	2815	1549	2872	1517
2	500	350	8	870.2	1137	640.3	1182	620.9
3	500	350	6	242.27	360.1	202.4	371.1	193.9
4	400	280	10	3101.5	4414	2432	4501	2359
5	400	280	8	1502.2	1808	1014	1841	970.2
6	400	280	6	508.68	572.1	317.8	584.3	3025
7	300	210	10	4928.5	7827	4320	7905	4102
8	300	210	8	2302	3214	1766	3256	1714
9	300	210	6	657.68	1012	564	1018	543.3
10	200	140	6	1601.5	2268	1251	2320	1217

 Table 10: Comparison between the results of (Johnson and ANSYS) with the Experimental work for intermediate columns after (SP+UIT)

Sp. No.	L (mm)	L _{eff} (mm)	D (mm)	P _{cr} Exper. (N)	P _{cr} Johnson (N)	P _{cr} Johnson S. F=2.4 (N)	Pcr ANSYS (N)	P _{cr} ANSYS S. F= 2.7 (N)
1	200	140	10	7035	15630	6514.9	17840	6604.8
2	200	140	8	3213.3	7330	3071.8	7313	2716.7

Tables (7, 8, 9, and 10) show that after using (SP+UIP) the critical buckling load (Pcr) of the columns was increased, this enhancement can

be clarified better by using bar charts for experimental results as shown in figures (9 and 10) below.



Figure 9. Illustrates the effect of (SP+UIP) on P_{cr} for long columns



Figure 10. Illustrates the effect of (SP+UIP) on P_{cr} for intermediate columns

Figures (7 and 8), illustrate that the critical buckling load (Pcr) of the specimens was increased due to the surface treatment processes used in the current work (SP+UIP).

6. Safety Factor (Design Factor) SF

The safety factor (SF) is determined for the cases tested using the following equation: S. F = Pcr Euler or Pcr Johnson or Pcr ANSYS / (Pcr)Experimental (6). Applying the above equation to Pcr Euler, Johnson and ANSYS gave the (SF) for both cases without and with (SP +UIP) as shown in Table (11) below:

Table 11: SF obtained from applying the buckling theories with ANSYS 18.2 (APDL)

SF					
Euler		Johnson		ANSYS 18.2 (APDL)	
Without (SP+UIP)	With (SP+UIP)	Without (SP+UIP)	With (SP+UIP)	Without (SP+UIP)	With (SP+UIP)
Long Columns					
1.8	1.8			2.1	1.8
Intermediate Columns					
		2.3	2.2	2.5	2.6

7. Conclusions

The following main conclusion can be drawn from the current study:

- 1. The combined approaches (SP +UIT) have the potential to significantly improve the mechanical properties and buckling behaviour ofAA2014-T4.
- The critical buckling load (Pcr) values were improved by the surface treatments (SP +UIP). An example of these improvements is the value of Pcr of the column with dimensions (L= 500mm, and D= 10mm) improved from (1560 Nto1717N) by (SP+UIP).
- 3. The results of the theories of Euler and Johnson were theories were compared with the experimental ones and good agreement was obtained with a factor of safety of (1.7) and (2.4 and 2.3) for Euler and Johnson before and after (SP+UIP).
- Numerical simulations (ANSYS 18.2 (APDL)) were also compared with the experimental data and gave excellent predictions with a factor of safety of (2.1 and 2.6) and (1.8 and 2.6) for long and medium columns with and without (SP +UIP), respectively.
- 5. The experimental mechanical results show a percentage improvement for UTS and YS of 2.64% and 3.17%, respectively, for the columns obtained due to (SP+UIT) techniques.
- 6. The advantages obtained by using a combination of techniques (SP +UIT) are the result of residual compressive stress.

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