



## Design and Implementation of Multistage Vector Quantization Algorithm of Image compression assistant by Multiwavelet Transform

Assist Instructor/ BASHAR TALIB HAMEED  
DIYALA UNIVERSITY, COLLEGE OF SCIENCE  
MSC\_BASHAR2000@YAHOO.CO.UK

### Abstract:

This paper presents a new coding technique based on contourlet transform and multistage vector quantization. Multiwavelet based Algorithms for image compression results in high compression ratios compared to other compression techniques. Multiwavelet have shown their ability in representing natural images that contain smooth areas separated with edges. However, wavelets cannot efficiently take advantage of the fact that the edges usually found in natural images are smooth curves.

This issue is addressed by directional transforms, known as contourlets, which have the property of preserving edges. The contourlet transform is a new extension to the Multiwavelet transform in two dimensions using nonseparable and directional filter banks.

The computation and storage requirements are the major difficulty in implementing a vector quantizer. In the full-search algorithm, the computation and storage complexity is an exponential function of the number of bits used in quantizing each frame of spectral information. The storage requirement in multistage vector quantization is less when compared to full search vector quantization. The coefficients of contourlet transform are quantized by multistage vector quantization. The quantized coefficients are encoded by Huffman coding to get better quality i.e., high peak signal to noise ratio (PSNR). The results obtained are tabulated and compared with the existing Multiwavelet based ones .

**Keywords:** Image compression, Multiwavelets, Multi-stage vector quantization.

## 1. Introduction

A fundamental goal of image compression [1] is to reduce the bit rate for transmission or data storage while maintaining an acceptable fidelity or image quality. Image compression is essential for applications such as TV transmission, video conferencing, facsimile transmission of printed material, graphics images, fingerprints and drawings. Compression can be achieved by transforming the data, projecting it on a basis of functions, and then encoding this transform. In this paper, we examine the design of image coder by integrating contourlet transform [2] with Multistage Vector Quantization (MSVQ) [3]. Vector quantization (VQ) is a quantization technique [4] applied to an ordered set of symbols. The superiority of VQ lies in the block coding gain, the flexibility in partitioning the vector space, and the ability to exploit intra-vector correlations. Multistage VQ divides the encoding task into several stages.

The first stage performs a relatively crude encoding of the input vector using a small codebook. Then, the second stage quantizer operates on the error vector between the original vector and the quantized first stage output. The quantized error vector provides a refinement to the first approximation. The indices obtained by multistage vector quantizer are then encoded using Huffman coding. Contourlets have the property of preserving edges and fine details in the image; the encoding complexity in multistage vector quantization is less when compared to tree structured vector quantization. This motivates us to develop a new coding scheme by integrating contourlet transform with multistage vector quantization. [5] [6].

In this paper, we propose a new image compression scheme for gray scale image compression using successive approximation quantization of vectors of the multiwavelet transformed image. This paper is organized as follows:

- 1- The concept of multiwavelet and multiwavelet filter banks.
- 2- The concept of non-linear approximation where the multiwavelet coefficient Are reordered, the significant transform coefficients of the image are retained and set the rest to zero .
- 3- Gives brief introduction to MSVQ. The proposed algorithm.

## 2. Multiwavelet

In multiwavelet transform, we use multiwavelet as transform basis. Multiwavelet functions are functions generated from one single function  $\psi$  by scaling and translation:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi_a\left(\frac{t-b}{a}\right) \quad \dots (1)$$

The mother wavelet  $\psi(t)$  has to be zero integral,  $\int \psi_{a,b}(t) dt = 0$ . From (1) we see that high frequency multiwavelet correspond to  $a > 1$  or narrow width, while low frequency multiwavelet corresponds to  $a < 1$  or wider width. The basic idea of wavelet transform is to represent any function  $f$  as a linear superposition of wavelets. Any such superposition decomposes  $f$  to different scale levels, where each level can be then further decomposed with a resolution adapted to that level. One general way to do this is writing  $f$  as the sum of wavelets  $\psi_{m,n}(t)$  over  $m$  and  $n$ . This leads to discrete wavelet transform:

$$f(t) = \sum_{m,n} \psi_{m,n}(t) \quad \dots (2)$$

By introducing the multi-resolution analysis (MRA) idea by Mallat [3], in discrete wavelet transform we really use two functions: wavelet function  $\psi(t)$  and scaling function  $\phi(t)$ . If we have a scaling function  $\phi(t) \in L^2(\mathbb{R})$ , then the sequence of subspaces spanned by its scaling and translations  $\psi_{j,k}(t) = 2^{j/2} \phi(2^j t - k)$ , i.e

$$V_j = \text{span} \{ \phi_{j,k}(t), k \in \mathbb{Z} \} \quad \dots (3)$$

Constitute a MRA for  $L^2(\mathbb{R})$ .

$\phi(t)$  must satisfy the MRA condition:

$$\phi(t) = \sqrt{2} \sum h(n) \phi(2t-2n) \quad \dots (4)$$

For  $n \in \mathbb{Z}$ . In this manner, we can span the difference between spaces  $V_j$  by wavelet functions produced from mother wavelet:  $\psi_{j,k}(t) = 2^{j/2} \phi(2^j t - k)$  Then we have:

$$\psi_{j,k}(t) = \sqrt{2} \sum g(n) \phi(2t-2n) \quad \dots (5)$$

For orthogonal basis we have:

$$g(n) = (-1)^n h(-n+1) \quad \dots (6)$$

If we want to find the projection of a function  $f(t) \in L^2(\mathbb{R})$  on this set of subspaces, we must express it in e as a linear combination of expansion functions of that subspace [4]:

$$f(t) = \sum_{n=-\infty}^{\infty} c(n) \phi(t) + \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} d(m) \psi_{j,k} \quad \dots (7)$$

Where  $\varphi_k(t)$  corresponds to the space  $V_0$  and  $\psi_{j,k}(t)$  corresponds to wavelet spaces. By using the idea of MRA implementation of wavelet decomposition can be performed using filter bank constructed by a pyramidal structure of lowpass filters  $h(n)$  and high pass filters  $g(n)$ [3, 4].

The matrix elements provide more degrees of freedom than a traditional scalar wavelet. These extra degrees of freedom can be used to incorporate useful properties into the multiwavelet filters, such as orthogonally, symmetry, and high order of approximation. The multiwavelet transform is implemented through a filter bank structure [7]. The multiwavelet decomposition of 'Lena' image is shown in Fig. 1. Unlike scalar wavelets, in which Mallat's pyramid algorithms [8] can be employed directly, the application of multiwavelets requires that the input signal first be vectorised namely preprocessing, this is popularly known as multiwavelet initialization or prefiltering [9]. In this paper, the preprocessing is based on Strela's algorithms [4], [10].



**Fig. 1 (a) First Level (b) Second Level Multiwavelets decomposition of Lena image**

A preprocessing scheme is described based on the approximation properties of the multiwavelets which yield a critically sampled image each of size  $(MXN)/4$ . Another advantage of this preprocessing is that it fits naturally with symmetric extension to multiwavelets.

### **3. Multi-Stage Vector Quantization**

Vector quantization is a powerful tool for data compression. Vector quantization extends scalar quantization to higher dimensional space. By grouping input samples into vectors and using a vector quantizer, a lower bit rate and higher performance can be achieved. However, the



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codebook size and the computational complexity increase exponentially as the rate increases for a given vector size. Full-search VQ such as entropy-constrained VQ (ECVQ) enjoys small quantization distortion. However, it has long compression time, and may not be well suited for real time signal compression systems. Tree-structured VQ (TSVQ) although can significantly reduce the compression time, has the disadvantage that the storage size required for the VQ is usually very large and cannot be controlled during the design process. Therefore, it may not be convenient to use TSVQ for the applications where the storage size is a major concern. A structured VQ scheme which can achieve very low encoding and storage complexity is MSVQ [12]. This appealing property of MSVQ motivated us to use MSVQ in the quantization stage. The basic idea of multistage quantization is to divide the encoding task into successive stages, where the first stage performs a relatively crude quantization of the input vector. Then a second-stage quantizer operates on the error vector between the original and the quantized first-stage output. The quantized error vector then provides a second approximation to the original input vector thereby leading to a refined or more accurate representation of the input. In this paper, we have implemented two-stage vector quantizer. The input vector is quantized by the initial or first stage vector quantizer denoted by VQ1 whose code book is  $C1 = c_{10}, c_{11} \dots c_{1(N1-1)}$  with size  $N1$ . The quantized approximation  $\hat{x}_1$  is then subtracted from  $x$  producing the error vector. This error vector is then applied to a second vector quantizer VQ2 whose code book is  $C2 = c_{20}, c_{21} \dots c_{2(N2-1)}$  with size  $N2$  yielding the quantized output. [13]

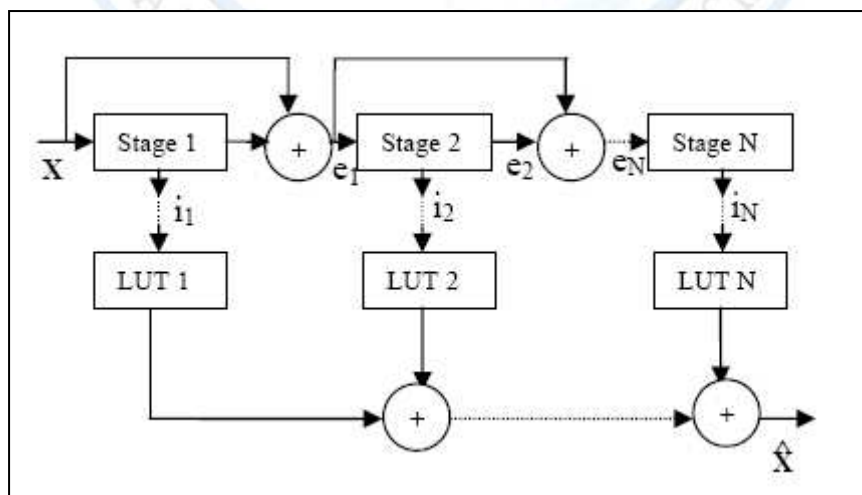


Fig. 2 Multistage Vector Quantization System

The encoder transmits a pair of indices specifying the selected codeword for each stage and the task of the decoder is to perform two table lookups to generate and then sum the two code words. In fact, the overall codeword or index is the concatenation of code words or indices chosen from each of two codebooks. Thus, the equivalent product codebook can be generated from the Cartesian product  $C_1 \times C_2$ . Compared to the full-search VQ with the product codebook  $C$ , the two stage VQ can reduce the complexity from  $N = N_1 \times N_2$  to  $N_1 + N_2$ . The multistage vector quantization system for 'N' stages is shown in Fig. 2. In the figure, 'X' represents the input vector, LUT stands for lookup table and  $i_1, i_2, \dots$  etc represent indices from different stages. The overall index is the concatenation of indices chosen from each of the two codebooks. From the Fig.2, it is evident that the input vector is given only to the first stage, whereas the input to the successive stages is the error vectors from the previous stage which are denoted by  $N \dots \dots e, e_2, \dots, e_1$ .  $X^{\wedge}$  is the reconstructed signal at the decoder end[6][12].

#### **4- MSVQ Encoder**

The MSVQ encoder, the input vector 'X' is quantized with the first stage codebook producing the first stage code vector  $Q_0(X)$ , a residual vector  $y_0$  is formed by subtracting  $Q_0(X)$  from 'X'. Then  $y_0$  is quantized using the second stage codebook, with exactly the same procedure as in the first stage, but with ' $y_0$ ' instead of 'X' as the input to be quantized. Thus, in each stage except the last stage, a residual vector is generated and passed to the next stage to be quantized independently of the other stages. MSVQ is an error refinement scheme, inputs to a stage are residual vectors from previous stage and they tend to be less and less correlated as the process proceeds[12].

#### **5- MSVQ Decoder**

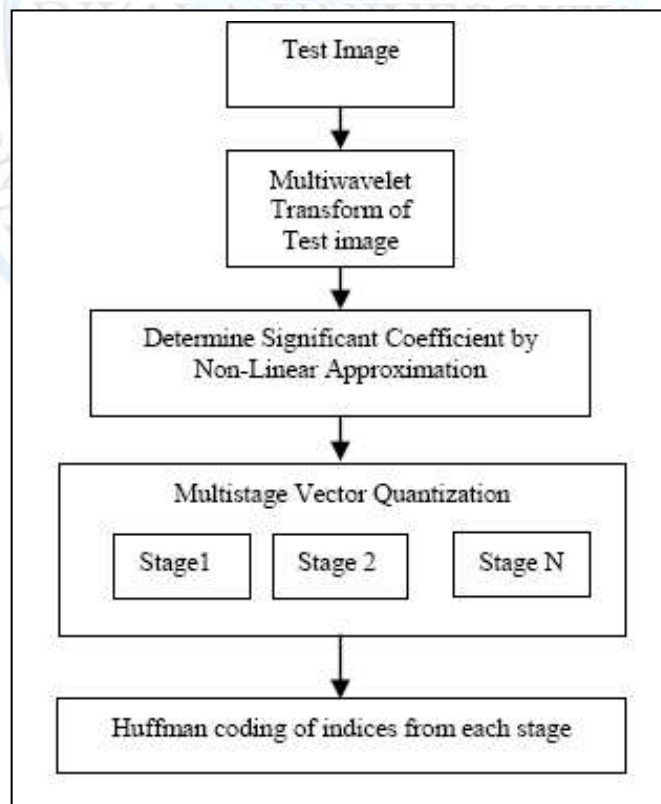
The decoder receives for each stage an index identifying the stage code vector selected and forms the reproduction  $X$  by summing the identified vectors. The overall quantization error is equal to the quantization residual from the last stage[12]. Sequential searching of the stage codebooks renders the encoding complexity to the storage complexity

$$\sum_{l=1}^L 2^{b_l}$$

## 6. PROPOSED ALGORITHM

The proposed image coder scheme is explained below.

1. The correlation present in the input image is removed by taking multiwavelet transform of the input image.
2. The non-linear approximation of the multiwavelet coefficients is performed.
3. The transform coefficients obtained in step 2 are vector quantized in a multistage manner where the residual error coefficients due to quantization are iteratively feedback and vector quantized. If the number of stages in MSVQ is more, the refinement of the quantized vectors will be better. But it suffers from the need for a high bit- rate for each additional stage is added. Hence we have restricted our attention to two stages in Multistage Vector Quantization.
4. The outputs from step 3 are lossless coded using static Huffman code. This completes the encoder stage of the proposed algorithm which is illustrated in Fig. 3. In decoding, the decoder basically performs the reverse process of the above steps.



**Fig. 3 Encoder of the proposed algorithm**

## **7. Results AND Discussion**

We present the encoding results for 256 X 256, 8 bit resolution 'Lena', 'Barbara' images [13]. 'Lena' is class of natural image that do not contain large amounts of high-frequency or oscillating patterns. 'Barbara' image exhibits large amounts of high-frequency and oscillating patterns.

The images are decomposed using multiwavelet transform. The multifilters used in this experiment are CL, BSA6/6, and GHM. The prefilter and post filter chosen are CL, BSA6/6, GHM, respectively. The encoded multiwavelet coefficients are subjected to non-linear approximation and the resultant coefficients are coded by multistage vector quantization. The results are compared with that of scalar wavelets in the same way. Table 1, 2 shows the comparative results of 'Lena', 'Barbara' images using 'CL' as multifilter and 'HAAR' as scalar filter. From these tables the following conclusions can be drawn

- (1) When the percentage of the significant coefficient retained is less, multiwavelet is giving better PSNR when compared to scalar wavelets.
- (2) As the level of decomposition increases, the performance of scalar wavelet matches with that of multiwavelet.
- (3) As the bit rate increases, the PSNR value increases which is in accordance with Rate-Distortion theory. Figure 4 shows the plot of PSNR against the percentage of coefficients retained for multiwavelets and wavelets with rate as two, under second level of decomposition. When the NLA coefficients retained is less the gap between the performance of multiwavelet and scalar wavelet is more and it merges with the increase in NLA coefficients. This is completely evident from the Fig. 4.



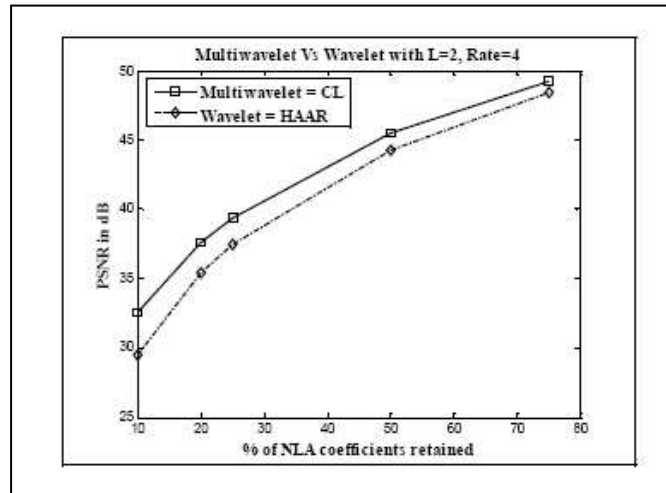


Fig. 4 Comparison of PSNR values for Multiwavelets and wavelets for ‘Lena’ image

Figure 5 shows the original and the reconstructed ‘Lena’ image using multiwavelet transform and wavelet transform with 10% of the coefficients retained with first level of decomposition and rate as four. From the Fig. 5b and 5c, it is obvious the visual quality of the reconstructed image using multiwavelet transform is better than that of wavelet transform. Figure 6 shows the original and reconstructed ‘Barbara’ image using wavelet and multiwavelet transform with 25% of the significant coefficient retained for the first level of decomposition and the selected rate is four.

We have used our algorithm to compare the performance of different multifilters against different wavelets like ‘HAAR’, ‘DB4’, for ‘Lena’ image with 25% NLA coefficients retained, and the results are tabulated in table 3. Figure 8, shows the plot of PSNR against the significant coefficients retained for multiwavelets and wavelets with the rate as eight under second level of decomposition.

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**TABLE 1**  
**PSNR RESAULT FOR ' LENA ' IMAGE**

Level of decomposition	Multiwavelet (PSNR in db)				Wavelet(PSNR in db)				
	10	20	25	50	10	20	25	50	
1	10	23.28	29.17	30.18	30.24	11.84	10.76	10.51	10.45
	20	24.71	33.81	36.56	36.83	19.56	19.94	19.29	19.44
	25	25.01	34.93	38.63	39.02	23.05	27.56	28.05	28.02
	50	22.67	37.40	45.42	47.21	25.32	36.51	43.26	44.47
	75	25.92	38.46	49.63	56.25	25.77	38.23	28.79	54.23
2	10	23.43	31.02	32.59	32.71	22.19	28.44	29.48	29.57
	20	24.22	34.14	37.60	37.95	23.23	32.48	35.42	35.72
	25	24.43	34.98	39.68	39.89	23.53	33.55	37.44	37.89
	50	24.96	37.00	45.51	47.64	24.29	36.11	44.29	46.26
	75	25.17	37.90	49.20	56.37	24.59	37.37	48.41	55.08
3	10	23.24	31.20	32.84	32.98	22.11	29.67	31.24	31.38
	20	23.92	34.13	37.68	38.08	22.91	32.65	36.08	36.44
	25	24.11	34.94	39.43	39.99	23.16	33.53	37.86	38.39
	50	24.58	36.86	45.38	47.66	23.84	35.84	44.28	46.44
	75	24.77	37.71	48.83	56.25	24.10	37.04	48.11	55.00

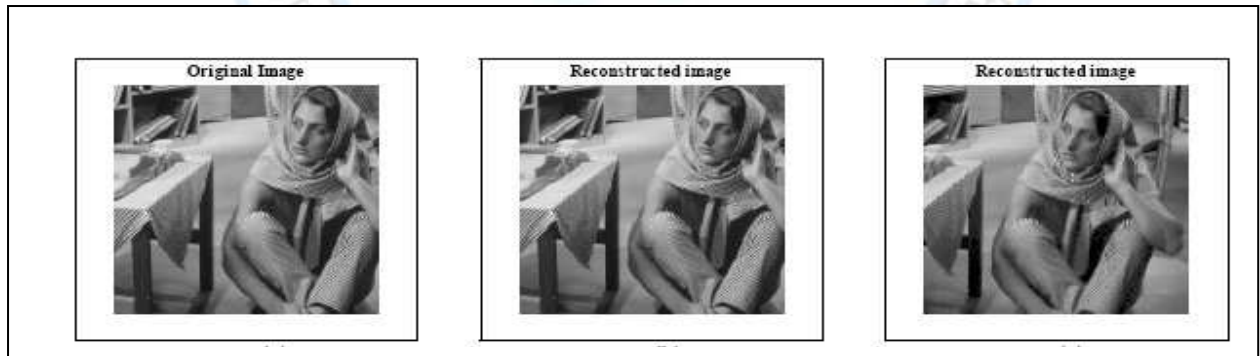


**Fig. 5 (a) Original image (b) Reconstructed image using multiwavelet transform (c) Reconstructed image using scalar wavelet**

**TABLE 2**  
**PSNR RESAULTA FOR ' BARARA ' IMAGE**

Level of decomposition	Multiwavelet (PSNR in db)				Wavelet(PSNR in db)				
	1	2	4	8	1	2	4	8	
1	10	21.60	25.06	25.39	25.41	12.53	11.46	11.20	11.14
	20	23.28	29.79	30.80	30.87	18.41	20.11	19.84	19.76
	25	23.71	31.46	33.01	33.12	22.20	24.12	24.16	24.13
	50	24.97	35.98	42.09	43.03	25.89	35.72	38.93	39.23
	75	25.48	37.88	48.33	53.23	26.23	38.72	48.27	51.48
2	10	21.73	26.71	27.32	27.36	21.22	24.41	24.70	24.72
	20	22.84	30.41	31.92	32.04	23.09	29.02	29.83	29.87
	25	23.23	31.81	33.95	34.13	23.61	30.77	32.01	32.09
	50	24.37	35.34	42.34	43.49	24.92	35.86	41.30	42.04
	75	24.82	37.29	47.94	53.35	25.42	37.77	47.97	52.26
3	10	21.65	36.84	27.51	27.56	21.66	25.73	26.15	26.17
	20	22.71	30.42	32.01	32.15	23.02	29.56	30.58	30.64
	25	32.10	32.79	34.02	34.22	23.47	31.18	32.62	32.72
	50	24.17	35.47	42.26	43.50	24.66	35.66	41.40	42.22

From the Coefficients retained for multiwavelets and wavelets with the rate as eight under second level of decomposition. From the figure it is evident that,

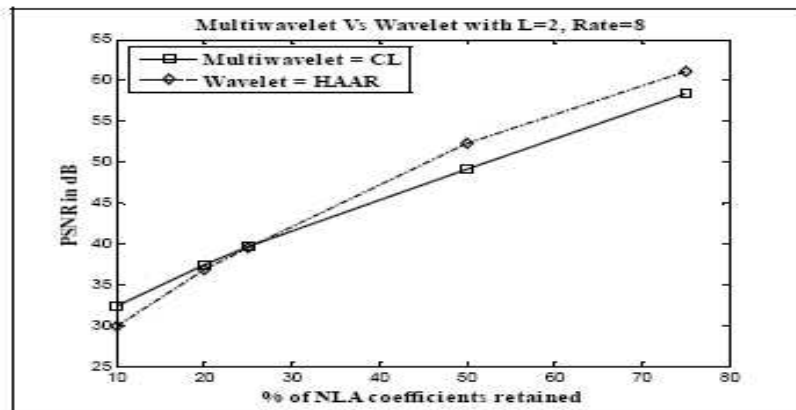


**Fig. 6 (a) Original image (b) Reconstructed image using multiwavelet transform (c) Reconstructed image using scalar wavelet**

**TABLE 3**  
**RESULTS OF THE MSOQ SCHEME FOR LENA IMAG**

Multiwavelet (PSNR in db)			Wavelet(PSNR in db)		
RATE	BSA6/6	CL	GHM	HAAR	DB4
1	26.2	24.90	23.18	23.20	23.06
2	34.89	34.95	28.23	29.28	27.56
4	38.63	38.90	28.41	29.74	28.07
6	39.02	39.40	28.80	29.77	28.04

when the percentage of significant coefficient retained is less than twenty percent, the performance of multiwavelet is better than scalar wavelet, at around thirty percent of retained coefficients, both multiwavelets and wavelets are performing equally well beyond that wavelets dominates multiwavelets.



**Fig. 8 Comparison of Multiwavelets against wavelets for ‘Boat’ image**

Figure 9 shows the performance of Multiwavelets tested for different images under third level of decomposition with twenty percentages of NLA coefficients retained, from the figure, it is evident that PSNR obtained in the case of ‘Lena’, is better than that of ‘Barbara’ image. From this we can conclude that the proposed scheme works well for low frequency images than high frequency image, under the same amount of NLA coefficients retained.



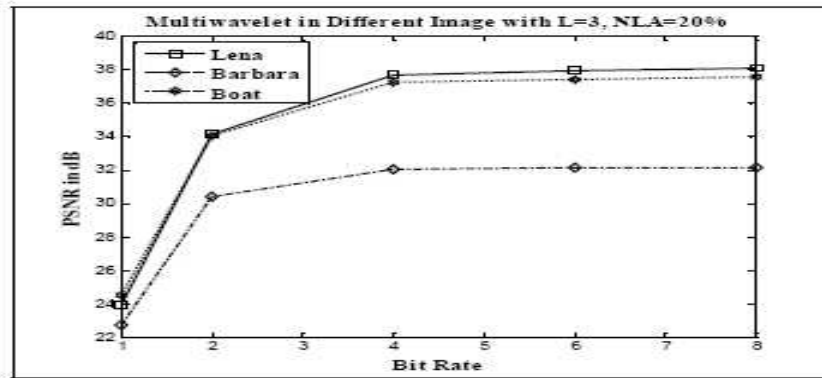


Fig. 9 Comparative performances of multiwavelets for different images

## 8. Conclusion

In this work we have proposed a new image coding algorithm based on non-linear approximation of multiwavelet transform along with multistage VQ. Our aim is to compare the performance of multiwavelets against scalar wavelets on different images along with the application of multistage vector quantization on both the schemes. When the number of significant coefficients is less than fifty percent, the performance of multiwavelet dominates wavelets irrespective of images chosen. This implies that, few significant multiwavelet coefficients are sufficient to reconstruct the image in a better manner than with the same significant wavelet coefficients. If we allow more significant coefficients, the performance of scalar wavelets dominates the performance of multiwavelets. This proves that multiwavelet cannot always substitute scalar wavelets with respect to image compression even though multiwavelets offer the advantages of combining symmetry, orthogonally, and short support, properties not mutually achievable with scalar two-band wavelet systems.

## 9. References

- [1] Rao, K. R., and Yip, "Discrete Cosine Transform: Algorithms, Advantages, Applications," Academic Press, 1990.
- [2] Daubechies, I. "Ten Lectures on Wavelets", Philadelphia, PA: SIAM, 2006, pp. 14-26.
- [3] Xia, X. G. J. S. Geronimo, D. P. Hardin, and B. W. Suter, "Design of prefilters for discrete multiwavelet transforms," IEEE Trans. Signal Process., vol. 44, Jan. 2003, pp. 25-35.
- [4] Strela, V. P. N. Hellers, G. Strang, P. Topiwala, and C. Heil, " The application of multiwavelet filter banks to image processing", IEEE Trans. Image Process., vol. 8, pp. 548-563, Apr. 2004
- [5] R. M. Gray, "Vector Quantization," IEEE ASSP magazine, vol. 1, No.2, Apr. 2004, pp. 4-29
- [6] Gersho, A. and R.Gray, "Vector Quantization and Signal Compression", Kluwer Academic Publishers, M.A, 2005, pp.320-333.
- [7] Vetterli, M. and G. Strang "Time-varying filter banks and multiwavelets," Sixth IEEE DSP workshop, Yosemite, 2004, pp.45-49.
- [8] Mallat, S.G. "A theory for multiresolution signal decomposition: The wavelet representation", IEEE Trans. Pattern Anal. Mech. Intell., vol. 11, Jul. 2008, pp. 674-693.
- [9] Miller, J. T. and C.C. Li, "Adaptive multiwavelet initialization", IEEE Trans. Signal Process., vol. 46, Dec. 2008, pp. 3282-3291.
- [10] Strela, V. "Multiwavelets: theory and application" PhD dissertation, Department of Mathematics, Massachusetts Institute of Technology, 2006, pp.122-126.
- [11] Albert Cohen, I. Daubechies, G. Guleryuz and Micheal T.Orchard, "On the importance of combining wavelet-based nonlinear approximation with coding strategies", IEEE Trans. Information Theory, vol. 48, July 2002, pp. 1895-1921.
- [12] Juang , B.H. and A. Gray, "Multiple stage vector quantization for speech coding", in Proc. IEEE Int. Conf. Acous., Speech, Signal processing (Paris, France), Apr. 2002, pp. 597-600.
- [13] Martin, M. B. "Applications of Multiwavelets to Image Compression", Master Thesis, Department of Electrical Engineering, Virginia Tech, 2008, pp.220-244.