Calculation The Energy Levels and Energy Bands(g, β, γ-bands) for Gd(A=140-160) Isotopes

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الملخص

في هذا البحث تم استعمال نموذج البوزونات المتفاعلة لحساب مستويات الطاقة وحزم الطاقة لنظائر نواة الكادلينيوم ذات العدد الكتلي (140−160) باستخدام البرنامج (IBSS1.for). هذا البرنامج يمكنه حساب اليم الذاتية والمتجهات الذاتية التي تستعمل في حساب مستويات الطاقة.وقد تم مقارنة النتائج النظرية مع النتائج العملية المتوفرة وقد أظهرت توافقا جيدا.

Abstract

In the present work the interacting boson model version one have been used to calculate the energy levels and energy bands (g, β, γ-bands) of Gadolinium Gd(A=140-160) isotopes by using the program (IBSS1.for).This program calculate the eigen values and eigen vectors. The results were compared with the experimental data and they were found in a good agreement.

Introduction

An atomic nucleus is the small, heavy, central part of atom consisting of a nucleons. It has two groups of like particles: protons and neutrons each of these groups is separately distributed over certain energy states, and they are held together by their mutal interactions which turn out to be very complicated in detail[1,2].

Nuclei have a certain time – independent properties such as mass, size, charge, intrinsic angular momentum, and certain time – dependent properties such as radioactivity and artificial transmutations (nuclear reactions). The nuclei also have excited states, whose energy is usually treated under the first class of properties [1].

Nuclei can excit in different energy levels corresponding to different arrangements of nucleons in their allowed states, a nucleus in an excited state normally remains there for a very short time. Often, it decays or becomes de-excited by emitting electromagnetic radiation in the form of γ-ray transitions to states with lower energies [2].

Gamma radiation is the same as any other type of electromagnetic radiation in which a changing in electric field induces a magnetic field and vice versa, the classification of the different processes leading to the emission of photons is based on conservation of angular momentum and parity. A photon carries away angular momentum of magnitude given by quantum number I, which can have integer value greater than zero(I=1,2,3,....), and the radiation field can have even or odd parity for a given I, depending on whether the radiation type is electric or magnetic [3].

Electric multipole radiation, denoted by (E1,E2,E3,....,EL) has parity (-1)L and magnetic multipole radiation (M1,M2,M3,....,ML) has parity (-1)L+1 , not all transitions are dominated by a single radiation type and we can have what are called mixed transitions.[2]

No complete theory exsist which fully describes the structure and behavior of complex nuclei based solely on knowledge of the force acting between nucleons, a successful model should be able to give a reasonable account of the properties which can be checked by experiment [4].

Many of models currently in use in nuclear physics some of these models are; the uniform particle model, the liquid- drop model, the shell model, the collective model and later on the interacting boson model which have four versions, in the present work we concerned with version one.

Iashello F. and Arima A. (1974)[5] had developed the interacting boson model which is suitable for describing the structure of intermediate and heavy nuclei, it is of considerable theoretical interest since it shows the dynamical symmetries of the nuclei.

The first version of interacting boson model (IBM-1) deals with the total number of bosons without distinction between the proton-proton bosons, neutron – neutron bosons. The neutron – proton bosons are excluded because the valence protons and valence neutrons occupy different major shells, so that the formation of proton – neutron bosons becomes very improbable[5,6].

The interacting boson model assumed that the particle configurations which are most important in shaping the properties of low – lying states. The particles are coupled together forming pairs of angular momentum 0 or 2. These pairs are treated as bosons. The pairs with angular momentum I=0 called s-bosons, while the pairs with angular momentum I=2 called d-bosons[5,6].

The dynamical symmetries in this model are depending on the main group called U(6) of unitary transformations in 6 dimensions and it has 36 generators. The next step is to identify all possible subgroups of full U(6) groups, a subgroups are generated by subsetting the generators of full groups, which happens to close under commutation . The analysis of this group leads to three subgroups called chains as follows[5→7].

1. The vibrational SU(5) chain
2. The rotational SU(3) chain
3. The γ-unstable O(6) chain

Forming 36 generators given above, the 25 operators are related to the generator of the group SU(5), the group of unitary transformation in 5 dimension and only d-boson are employed by these operators, the next step 10 operators from 25 operators closed under the group O(5) the orthogonal group in 5 dimension, 3 operators from 10 of O(5) consist O(3) group which in 3 dimension and finally 1 operator closed under O(2).Thus, one possible chain of subgroups of the vibrational SU(5) chain is as follows[5, 7].

$$U(6) \supset SU(5) \supset O(5) \supset O(3) \supset O(2)$$

The rotational SU(3) group, which is the second group form of 8 operator from 36 operators of U(6) group in 3 dimensions , as in chain one 3 operators form the O(3) group, and this leads to O(2) which has 1 operator from O(3) operators. The SU(3) symmetry describes nuclei which are permanently deformed for the two types (prolate and oblate),this chain is[5→7].

$$U(6) \supset SU(3) \supset O(3) \supset O(2)$$

At last 15 operators built the O(6) group from U(6) group, the orthogonal in 6 dimensions, the second step that 10 operators from O(6) operator consists of the O(5) group, and like the above chain 3 operators form O(3) group in three dimensions, and 1 operator closed under O(2) group. Thus, a third possible chain is [4,5].

$$U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2)$$

Energy spectrum which gives the vibrational states and rotational states of excited nuclei depends on the intrinsic motion of these nuclei, these states named collective states (low – lying states). Many nuclei exhibit regular features that vary rather smoothly over wide region of A and do not depend significantly on number of valance nucleons or the particular shells they occupy[4].

The vibrational states assumed that the nucleus will be spherical in its ground state; it is possible for nucleus to vibrate about its equilibrium shape and exsist in quantum states. These states can be classified by considering the different vibrational modes of a liquid drop[3,4].

The rotational states can only be observed in nuclei with a non – spherical shape. Only if there is a deviation from spherical symmetry the rotational modes can be detected. For this reason, a nucleus shaped like an ellipsoid can rotate about one of its major or minor axis either parallel or perpendicular to the symmetric axis, i.e. rotational axis. The nucleus dose not rotate like a rigid body and only part of its nucleons can be considered to be involved in collective motion i.e., the nucleus is soft in the sense that its internal structure can change in response to the internal stresses that arise due to the high rate of rotation.[3]

It is well established that many nuclei with N and Z values between magic numbers are permanently deformed in their shape, the deformation arises because of the way valence nucleons arrange themselves in an unfilled shell, in other words the deformation happens only when both proton and neutron shells are partially filled[4].

There are two types of deformation depending on two facts[1,3,4]:

1. All deformed nuclei have quadrupole moments in their states.
2. The changing in the shape which depends on the direction of motion , which is classified into the following:

A- Oblate type; it has posative quantity of quadrupole moment; the direction of motion is parallel with the symmetric axis.

B- Prolate type; it has negative quantity of quadrupole moment and the direction of motion is perpendicular to the symmetric axis.

At last deformed nuclei are found throughout the periodic table and are most common in the mass region 150<A<190 and A>230. In the first region protons and neutrons are filling the shell above Z=50 and N=82 while the second, shells above Z=82 and N=126 are being filled[4].

The theoretical considerations

Boson operators in (IBM – 1)

In this model, it is assumed that the low – lying collective states of nuclei can be described in monopole boson with angular momentum and parity, $I^{\pi} = 0^{+}$, which is called s – boson and quadrupole boson with $I^{\pi} = 2^{+}$, which is calledd – boson, thus, the building of this model is [5,7] : -

$$\hat{s}^{\dagger}, \hat{d}^{\dagger}_{\mu} \qquad \hat{s}, \hat{d}_{\mu} \qquad \mu = 0, \mp 1, \mp 2, \ldots\ldots \qquad \textbf{.........(1)}$$

These operators satisfy the commutation relations [5]: -

$$[\hat{s}, \hat{s}^{\dagger}] = 1 \qquad ; \qquad [\hat{s}, \hat{s}] = [\hat{s}^{\dagger}, \hat{s}^{\dagger}] = 0$$

$$[\hat{d}, \hat{d}^{\dagger}] = 1 \qquad ; \qquad [\hat{d}, \hat{d}] = [\hat{d}^{\dagger}, \hat{d}^{\dagger}] = 0 \qquad \textbf{.........(2)}$$

The operators $\hat{\tilde{s}}$ and $\hat{\tilde{d}}_{\mu}$ are given by the relation [7]:-

$$\hat{\tilde{s}} = \hat{s}$$

$$\hat{\tilde{d}}_{\mu} = (-1)^{\mu}\hat{d}_{\mu} \quad \text{.........(3)}$$

The IBM – 1 Hamiltonian operator:

The Hamiltonian operator can be written as [5,6]: -

$$\hat{H} = \varepsilon\hat{n}_d + a_0\left(\hat{P}^{\dagger}.\hat{P}\right) + a_1\left(\hat{I}.\hat{I}\right) + a_2\left(\hat{Q}.\hat{Q}\right) + a_3\left(\hat{T}_3.\hat{T}_3\right) + a_4\left(\hat{T}_4.\hat{T}_4\right) \quad \text{.........(4)}$$

The six independent free parameters (ε, a_0, a_1, a_2, a_3, a_4) are fitted to the data corresponding to the dynamical symmetry according to the operators, and the operators $\hat{n}_d$, $\hat{P}$, $\hat{I}$, $\hat{Q}$, $\hat{T}_3$, $\hat{T}_4$ can be written as follows [5,6]:-

$$\left.\begin{aligned}
\hat{n}_d &= (\hat{d}^{\dagger}, \hat{\tilde{d}}) \\
\hat{P} &= \frac{1}{2}(\hat{\tilde{d}}.\hat{\tilde{d}}) - \frac{1}{2}(\hat{\tilde{s}}.\hat{\tilde{s}}) \\
\hat{I} &= \sqrt{10}\,[\hat{d}^{\dagger}, \hat{\tilde{d}}]^{1} \\
\hat{Q} &= [\hat{d}^{\dagger} \times \hat{\tilde{s}} + \hat{s}^{\dagger} \times \hat{\tilde{d}}]^{2} - \chi[\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{2} \\
\hat{T}_3 &= [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{3} \\
\hat{T}_4 &= [\hat{d}^{\dagger} \times \hat{\tilde{d}}]^{4}
\end{aligned}\right\} \quad \text{.........(5)}$$

Where

$\hat{n}_d$ =d – boson operator
$\hat{P}$ = pairing operator
$\hat{I}$ = angular momentum operator
$\hat{Q}$ = quadrupole operator

χ = -√7/2 for deformed nuclei

$\hat{T}_3$ = octapole operator

$\hat{T}_4$ = hexadeapole operator

and N is the total number of bosons defined as[5]: -

$$N = n_s + n_d \qquad \text{.........(6)}$$

So that

$$\varepsilon_s(\hat{s}^\dagger, \hat{\tilde{s}}) = \varepsilon_s \hat{\tilde{n}}_s = \varepsilon_s(\hat{N} - \hat{\tilde{n}}_d) = \varepsilon_s \hat{N} - \varepsilon_s \hat{\tilde{n}}_d \qquad \text{.........(7)}$$

The lie algebra U(6)

if we have the operator [5]: -

$$\hat{G}^k_\mu(I, I') = [\hat{b}^\dagger_l . \hat{\tilde{b}}_{l'}] \qquad \text{.........(8)}$$

Where I,Í = 0,2= s, d

The operator $\hat{G}^k_\mu$ is called generators of corresponding group. Their explicit expression in terms of s – boson and d – boson have 36 generators of U(6) which can be written as follows [5,6]: -

$$\left.\begin{aligned}
&\hat{G}^0_0(s,s) = [\hat{s}^\dagger \times \hat{\tilde{s}}]^{(0)}_0 \\
&\hat{G}^0_0(d,d) = [\hat{d}^\dagger \times \hat{\tilde{d}}]^{(0)}_0 \\
&\hat{G}^1_\mu(d,d) = [\hat{d}^\dagger \times \hat{\tilde{d}}]^{(1)}_\mu \\
&\hat{G}^2_\mu(d,d) = [\hat{d}^\dagger \times \hat{\tilde{d}}]^2_\mu \\
&\hat{G}^3_\mu(d,d) = [\hat{d}^\dagger \times \hat{\tilde{d}}]^2_\mu \\
&\hat{G}^4_0(d,d) = [\hat{d}^\dagger \times \hat{\tilde{d}}]^4_\mu \\
&\hat{G}^2_\mu(d,s) = [\hat{d}^\dagger \times \hat{\tilde{s}} + \hat{s}^\dagger \times \hat{\tilde{d}}]^2_\mu \\
&\hat{G}^2_\mu(s,d) = [\hat{s}^\dagger \times \hat{\tilde{d}} + \hat{d}^\dagger \times \hat{\tilde{s}}]^2_\mu
\end{aligned}\right\} \qquad \text{.........(9)}$$

The µ – tensor has (2k + 1) independent components of U (6)

The sub algebra groups

In interacting boson model U(6) group give three subgroups called chains as follows[5,6]: -

1. SU(5) vibrational chain.
2. SU(3) rotational chain.
3. O(6) γ – unstable chain.

Table (2 – 1) shows the chains analysis, the number of independent tensor component and their behavior.

Table (1): chains analysis, generators, independent component and behaviors [5,6]

chain	The chain analysis	generators	Independent tensor component	behavior
SU(5)	$SU(5) \supset O(5) \supset O(3) \supset O(2)$	$\hat{G}^0_0(d,d)$ $\hat{G}^1_\mu(d,d)$ $\hat{G}^2_\mu(d,d)$ $\hat{G}^3_\mu(d,d)$ $\hat{G}^4_\mu(d,d)$	25	Vibrational
SU(3)	$SU(3) \supset O(3) \supset O(2)$	$\hat{G}^0_0(d,d)$ $\hat{G}^0_0(s,s)$ $\hat{G}^2_\mu(s,d)$ $\hat{G}^2_\mu(d,s)$ $\hat{G}^1_\mu(d,d)$	9	Rotational
O(6)	$O(6) \supset O(5) \supset O(3) \supset O(2)$	$\hat{G}^1_\mu(d,d)$ $\hat{G}^2_\mu(s,d)$ $\hat{G}^2_\mu(d,s)$ $\hat{G}^3_\mu(d,d)$	15	γ -unstable

Transitional regions dynamical symmetries

A) SU(3) – SU(5)

In this transitional region the SU(3) symmetry has to be broken with an $\varepsilon\hat{n}_d$ term. The using of schematic Hamiltonian can be written as [5,6]: -

$$\hat{H} = \varepsilon\hat{n}_d + a_1(\hat{I}.\hat{I}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad \text{.........(10)}$$

Since the term $\hat{I}.\hat{I}$ is a Casmir invariant of both SU(3) and SU(5), the properties of the solutions of equation (10) depend only on the ratio ε/a_2.This ratio takes the role of a control parameter for this transitional region. When ε/a_2 is large, the Eigen functions of $\hat{H}$ are those appropriate to the limiting situation SU(5), while, when ε/a_2 is small, the Eigen functions of $\hat{H}$ are those appropriate to the limiting situation SU (3).

B) SU(3) – O(6)

Breaking SU (3) symmetry in the direction of O(6) in this transitional region can be treated by adding $(\hat{P}^{\dagger}.\hat{P})$ term. The Hamiltonian can be written in the form [5,6]: -

$$\hat{H} = a_0(\hat{P}^{\dagger}.\hat{P}) + a_1(\hat{I}.\hat{I}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) \quad \text{.........(11)}$$

When all terms are kept in equation (11), it appears that the situation in practice is controlled by the ratio a_0/a_2, when this is large, the Eigen functions of $\hat{H}$ are those appropriate to the limiting situation O (6), when it is small, the Eigen functions of $\hat{H}$ are those appropriate to the limiting situation SU (3). In this region, the γ – band will be lower than the β – band.

C) O(6) – SU(5)

The nuclei in this transitional region can be treated by Hamiltonian containing $\varepsilon\hat{n}_d$ and a_0 $(\hat{P}^{\dagger}.\hat{P})$ terms as [5,6]: -

$$\hat{H} = \varepsilon\hat{n}_d + a_0(\hat{P}^{\dagger}.\hat{P}) + a_1(\hat{I}.\hat{I}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad \text{.........(12)}$$

The control parameter here is ε/a_0.when it is large, the Eigen functions of $\hat{H}$ are those appropriate to the limiting situation SU(5). But when it is small, the Eigen functions of $\hat{H}$ are those appropriate to the limiting situation O(6).

Results and the discussions

The aim of this work is to study the nuclear structure and to predict the dynamical symmetries of deformed Gd (A=140-160) and calculation the energy levels for (g,β,γ – band).

The parameters of equation (4) fitted to the data are used to calculate Eigen values and Eigen vectors of Gd(A=140-160) isotopes which are tabulated in table (2).

Table (2): The parameter of Hamiltonian function operator for Gd(A=140-160) isotopes.

Isotopes	N_π	N_ν	N_{Tot}	ESP (MeV)	$\hat{P}^\dagger.\hat{P}$ (MeV)	$\hat{I}.\hat{I}$ (MeV)	$\hat{Q}.\hat{Q}$ (MeV)	$\hat{T}_3.\hat{T}_3$ (MeV)	$\hat{T}_4.\hat{T}_4$ (MeV)	CHI
${}^{140}_{64}Gd_{76}$	7	3	10	0.6600	0.0000	0.0050	-0.0150	-0.0010	-0.0450	-1.0330
${}^{142}_{64}Gd_{78}$	7	2	9	0.6600	0.0000	0.0150	-0.0150	0.0000	0.0000	-1.0230
${}^{144}_{64}Gd_{80}$	7	1	8	0.9000	0.0000	0.0190	-0.0190	0.0000	0.0020	-1.2230
${}^{146}_{64}Gd_{82}$	7	0	7	1.8000	0.0000	-0.0015	-0.0150	-0.0550	-0.0900	-1.3330
${}^{148}_{64}Gd_{84}$	7	1	8	0.8500	0.0000	-0.0040	-0.0010	0.0000	-0.0500	-1.0230
${}^{150}_{64}Gd_{86}$	7	2	9	0.5500	0.0100	-0.0055	-0.0001	-0.0050	-0.0450	-1.0220
${}^{152}_{64}Gd_{88}$	7	3	10	0.0000	0.0100	-0.0140	-0.0110	-0.0050	-0.0050	-1.9520
${}^{154}_{64}Gd_{90}$	7	4	11	0.0000	0.0015	0.0090	-0.0098	0.0650	0.0050	-1.5830
${}^{156}_{64}Gd_{92}$	7	5	12	0.0000	0.0001	0.0094	-0.0150	0.0050	0.0000	-1.0220
${}^{158}_{64}Gd_{94}$	7	6	13	0.0000	0.0090	0.0092	-0.0010	0.0000	0.0000	-1.8530
${}^{160}_{64}Gd_{96}$	7	7	14	0.0000	0.0090	0.0092	-0.0010	0.0000	0.0000	-1.2532

Table (3) listed the energy levels of Gd(A=140-160) isotopes understudy according to energy band (g,β,γ – bands).

In this table we notice that, our result for g,β,γ – bands was good agreement with the experimental data except in high spin. This differences in these three bands are very clear in figures (1),(2),(3), (4) and (5).

If the dynamical symmetry of the isotopes is O(6) then the γ – band precede the β – band in energy levels, i.e make braking to the main dynamical symmetry with the $\left(\hat{P}^\dagger.\hat{P}\right)$ term, which was not seen in our result, while the dynamical symmetries of Gd(A140-160) show mixture of SU(5)- SU(3), SU(5)-SU(3)-O(6) and SU(3)-O(6) dynamical symmetry.

Table (3): Theoretical (PW) and experimental energy levels in (MeV) of g, β, γ-bands for Gd(A=140-160) isotopes.

Isotopes	behavior	g,β,γ – bands[8,9,10] (MeV)						
		bands	$0^+(2^+)$	$2^+(3^+)$	$4^+(4^+)$	$6^+(5^+)$	$8^+(6^+)$	$10^+(7^+)$
$^{140}_{64}Gd_{76}$	U(5) – SU(3)	g(Theo.)	0.000	0.247	0.642	1.179	1.848	2.643
		g(Exp.)	0.000	0.329	0.837	1.513	2.189	2.846
		β(Theo.)	0.481	0.798	1.282	1.881	2.598	3.428
		β(Exp.)	-	0.713	1.281	1.881	2.846	2.976
		γ(Theo.)	1.007	1.287	1.538	1.837	2.154	2.502
		γ(Exp.)	-	1.069	-	1.694	-	2.412
$^{142}_{64}Gd_{78}$	U(5) – SU(3)	g(Theo.)	0.000	0.496	1.178	2.054	2.120	4.374
		g(Exp.)	0.000	0.526	1.248	2.081	2.927	-
		β(Theo.)	0.879	1.114	1.900	2.859	3.995	5.312
		β(Exp.)	-	-	-	-	-	-
		γ(Theo.)	1.570	1.789	2.415	2.680	3.429	3.748
		γ(Exp.)	-	-	-	-	-	-
$^{144}_{64}Gd_{80}$	U(5) – SU(3)	g(Theo.)	0.000	0.743	1.708	2.910	2.909	4.350
		g(Exp.)	0.000	0.743	1.743	2.862	-	-
		β(Theo.)	1.293	1.620	2.714	4.025	5.559	-
		β(Exp.)	-	-	-	-	-	-
		γ(Theo.)	2.265	2.587	3.4336	3.813	4.813	5.259
		γ(Exp.)	-	-	-	-	-	-
$^{146}_{64}Gd_{82}$	U(5) – SU(3)	g(Theo.)	0.000	1.386	2.786	4.208	5.662	-
		g(Exp.)	0.000	1.972	2.615	-	-	-
		β(Theo.)	2.598	2.715	4.089	5.486	6.913	
		β(Exp.)	2.165	3.378	-	-	-	-
$^{148}_{64}Gd_{84}$	U(5) – SU(3)	g(Theo.)	0.000	0.725	1.420	2.084	2.719	3.324
		g(Exp.)	0.000	0.785	1.416	1.811	2.693	-
		β(Theo.)	1.320	1.427	2.096	2.735	3.343	-
		β(Exp.)	-	1.463	-	-	-	-
		γ(Theo.)	1.948	2.099	2.547	2.741	3.116	3.352

		γ(Exp.)	-	-	-	-	-	-
$^{150}_{64}Gd_{86}$	U(5) – SU(3) – O(6)	g(Theo.)	0.000	0.637	1.234	1.790	2.305	2.780
		g(Exp.)	0.000	0.638	1.289	1.947	2.554	-
		β(Theo.)	1.224	1.259	1.829	2.359	2.848	-
		β(Exp.)	1.207	1.430	1.988	2.116		-
		γ(Theo.)	1.791	1.799	2.317	2.381	2.811	2.877
		γ(Exp.)	1.518	1.988	-	-	-	2.392
Sotopes	behavior	g,β,γ – bands[8,9,10] (MeV)						
		bands	$0^+(2^+)$	$2^+(3^+)$	$4^+(4^+)$	$6^+(5^+)$	$8^+(6^+)$	$10^+(7^+)$
$^{152}_{64}Gd_{88}$	U(5) – SU(3) – O(6)	g(Theo.)	0.000	0.208	0.615	1.211	1.990	2.947
		g(Exp.)	0.000	0.344	0.755	1.227	1.747	2.300
		β(Theo.)	0.427	0.781	1.302	1.988	2.842	-
		β(Exp.)	0.615	0.931	1.282	1.668	2.139	-
		γ(Theo.)	1.066	1.353	1.615	1.936	2.300	2.685
		γ(Exp.)	1.109	1.434	1.550	1.862	1.998	2.395
$^{154}_{64}Gd_{90}$	SU(3) – O(6)	g(Theo.)	0.000	0.109	0.345	0.697	1.159	1.731
		g(Exp.)	0.000	0.123	0.371	0.718	1.145	1.637
		β(Theo.)	0.566	0.708	0.979	1.379	1.909	2.568
		β(Exp.)	0.680	0.816	1.048	1.366	1.757	2.194
		γ(Theo.)	0.903	1.129	1.272	1.560	1.736	2.110
		γ(Exp.)	0.996	1.128	1.264	1.433	1.607	1.810
$^{156}_{64}Gd_{92}$	SU(3) – O(6)	g(Theo.)	0.000	0.092	0.307	0.644	1.103	1.681
		g(Exp.)	0.000	0.089	0.288	0.585	0.965	1.416
		β(Theo.)	0.939	0.953	1.173	1.517	1.982	2.565
		β(Exp.)	1.050	1.129	1.298	1.540	1.848	-
		γ(Theo.)	1.032	1.048	1.249	1.331	1.589	1.739
		γ(Exp.)	1.154	1.248	1.356	1.507	1.645	-
$^{158}_{64}Gd_{94}$	SU(3) – O(6)	g(Theo.)	0.000	0.078	0.261	0.549	0.941	1.373
		g(Exp.)	0.000	0.080	0.261	0.539	0.904	1.351
		β(Theo.)	0.983	1.091	1.272	1.558	1.948	-
		β(Exp.)	1.196	1.260	1.407	-	-	-
		γ(Theo.)	1.061	1.169	1.244	1.402	1.530	1.739
		γ(Exp.)	1.187	1.265	1.358	1.481	-	-
$^{160}_{64}Gd_{96}$	SU(3) – O(6)	g(Theo.)	0.000	0.077	0.256	0.538	0.923	1.410
		g(Exp.)	0.000	0.075	0.249	0.541	0.868	-
		β(Theo.)	0.898	0.975	1.155	1.437	1.822	2.310
		β(Exp.)	-	1.010	1.185	-	-	-

		γ(Theo.)	0.843	0.920	1.023	1.151	1.305	1.484
		γ(Exp.)	0.989	1.058	1.149	-	-	-

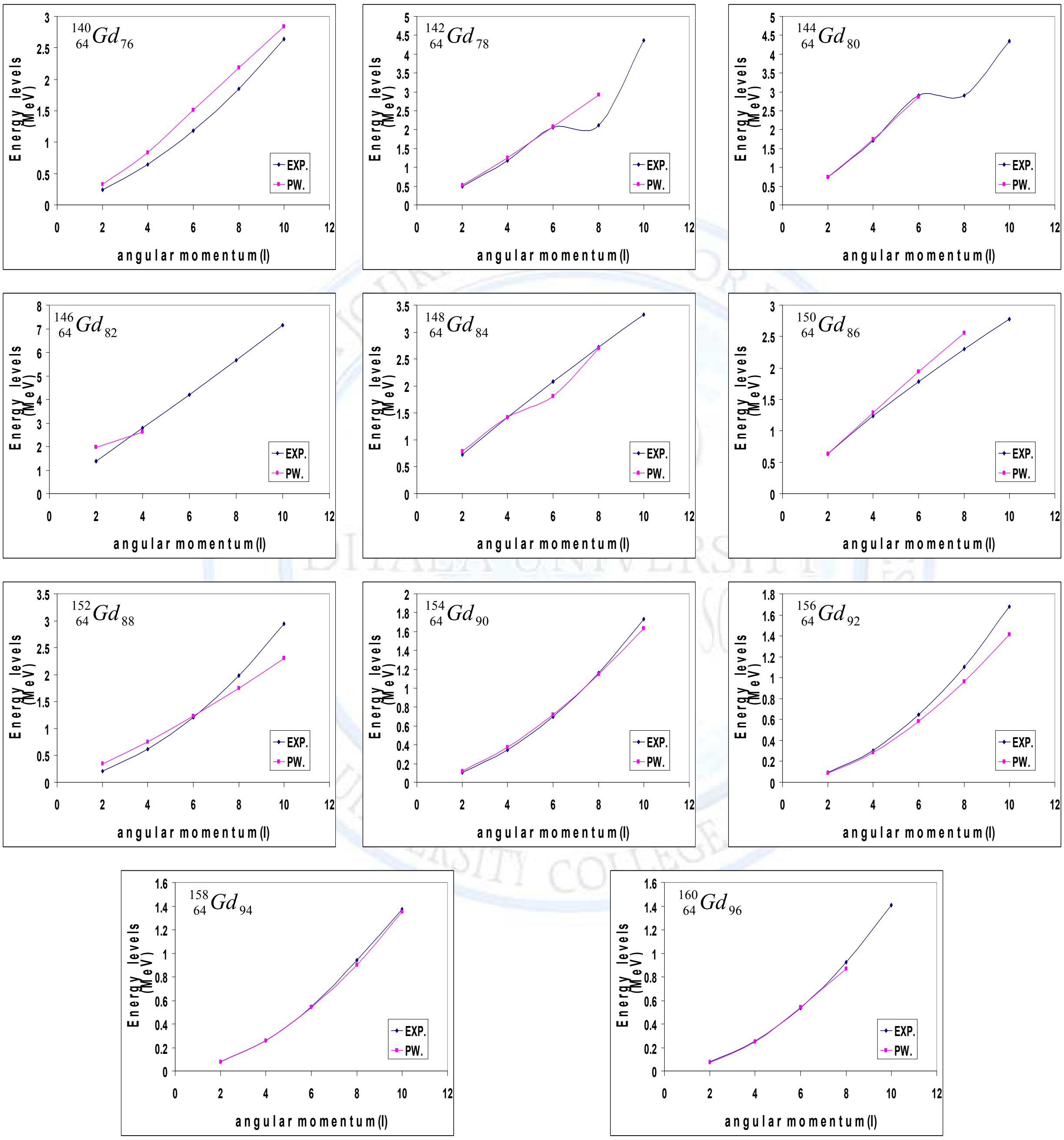

Figure (1): Theoretical (PW) and experimental energy levels versus angular momentum of g-bands for Gd (140-160) isotopes.

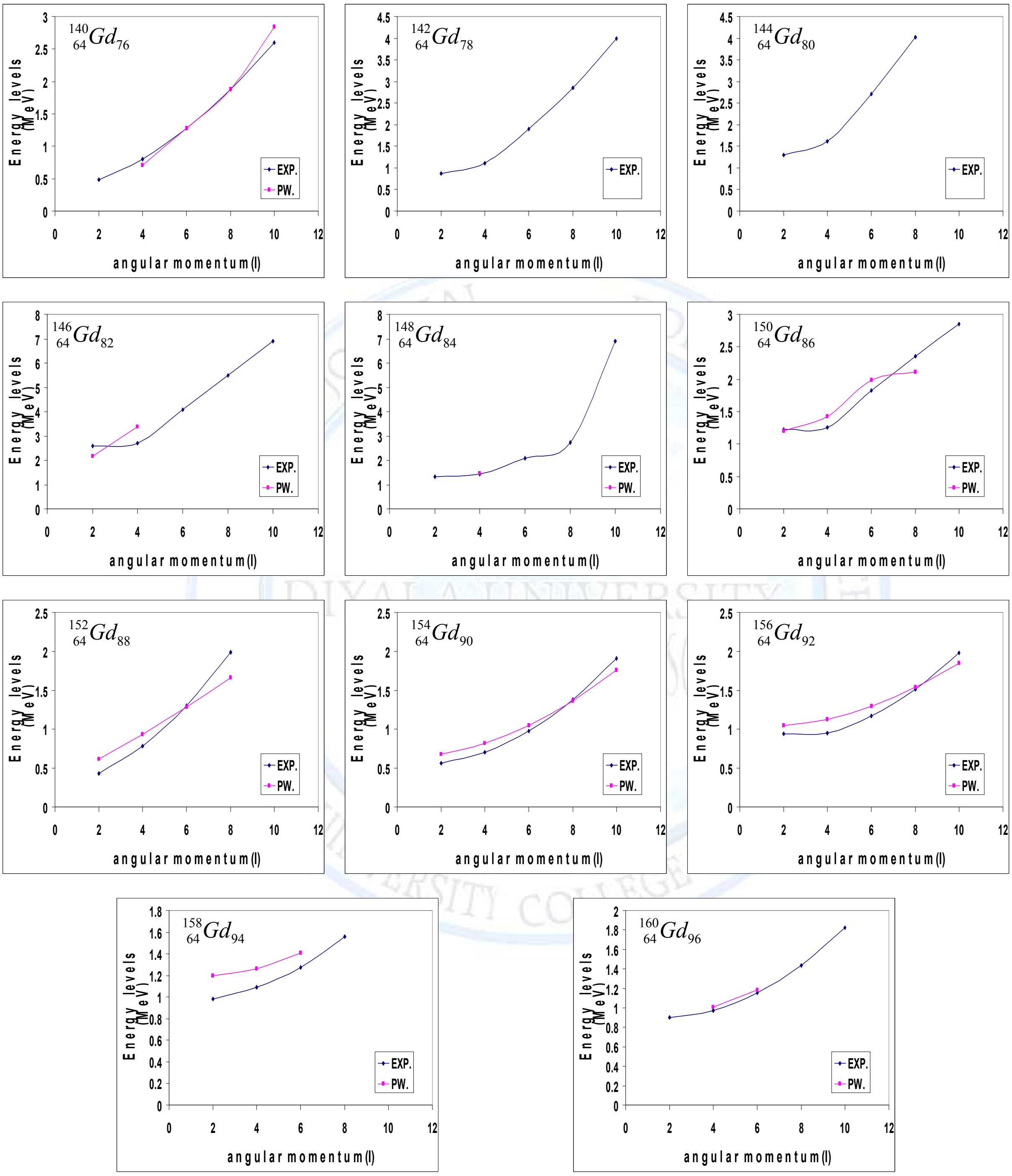

Figure (2): Theoretical (PW) and experimental energy levels versus angular momentum of β-bands for Gd(140-160) isotopes.

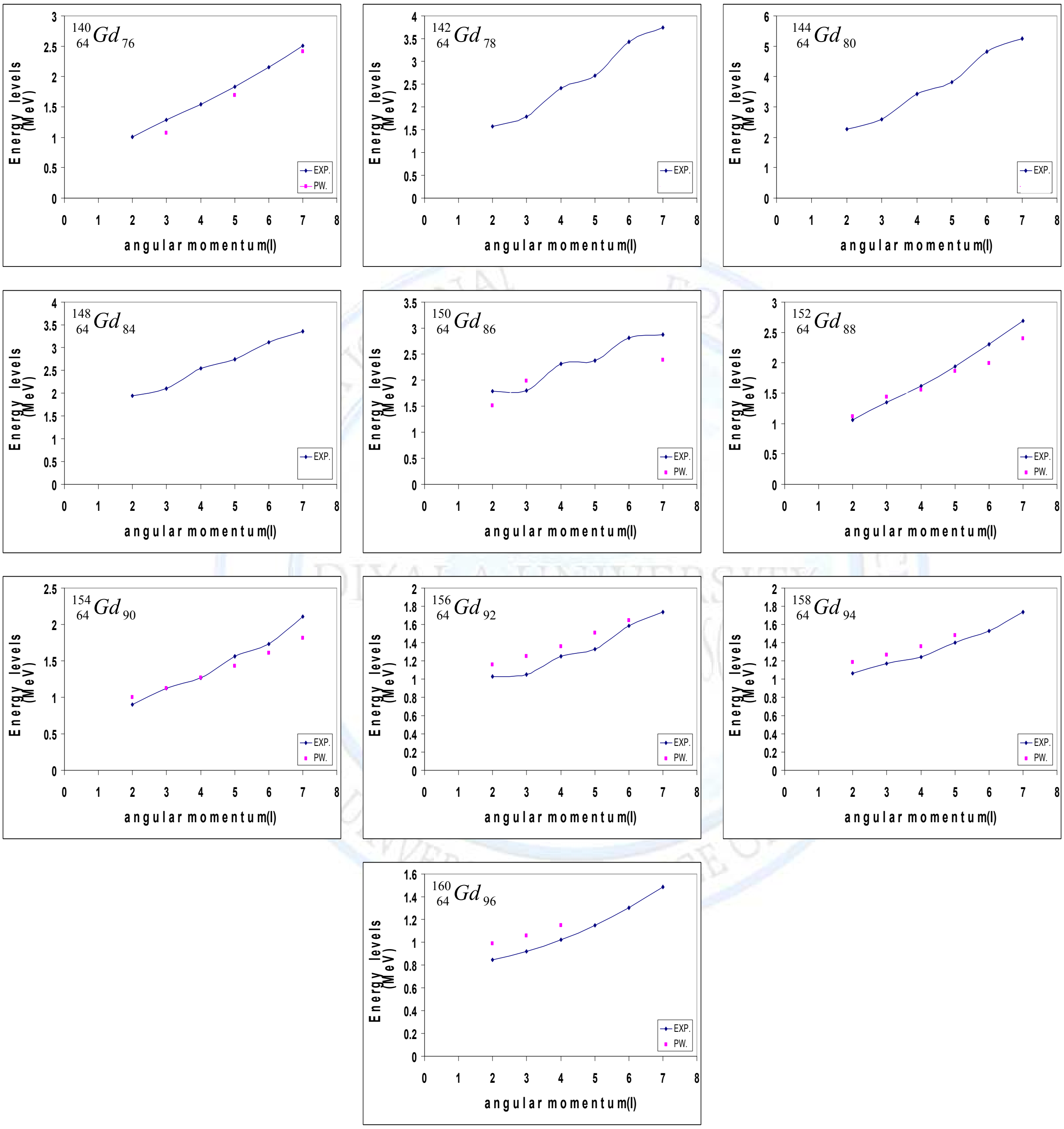

Figure (3): Theoretical (PW) and experimental energy levels versus angular momentum of γ-bands for Gd (140-160) isotopes.

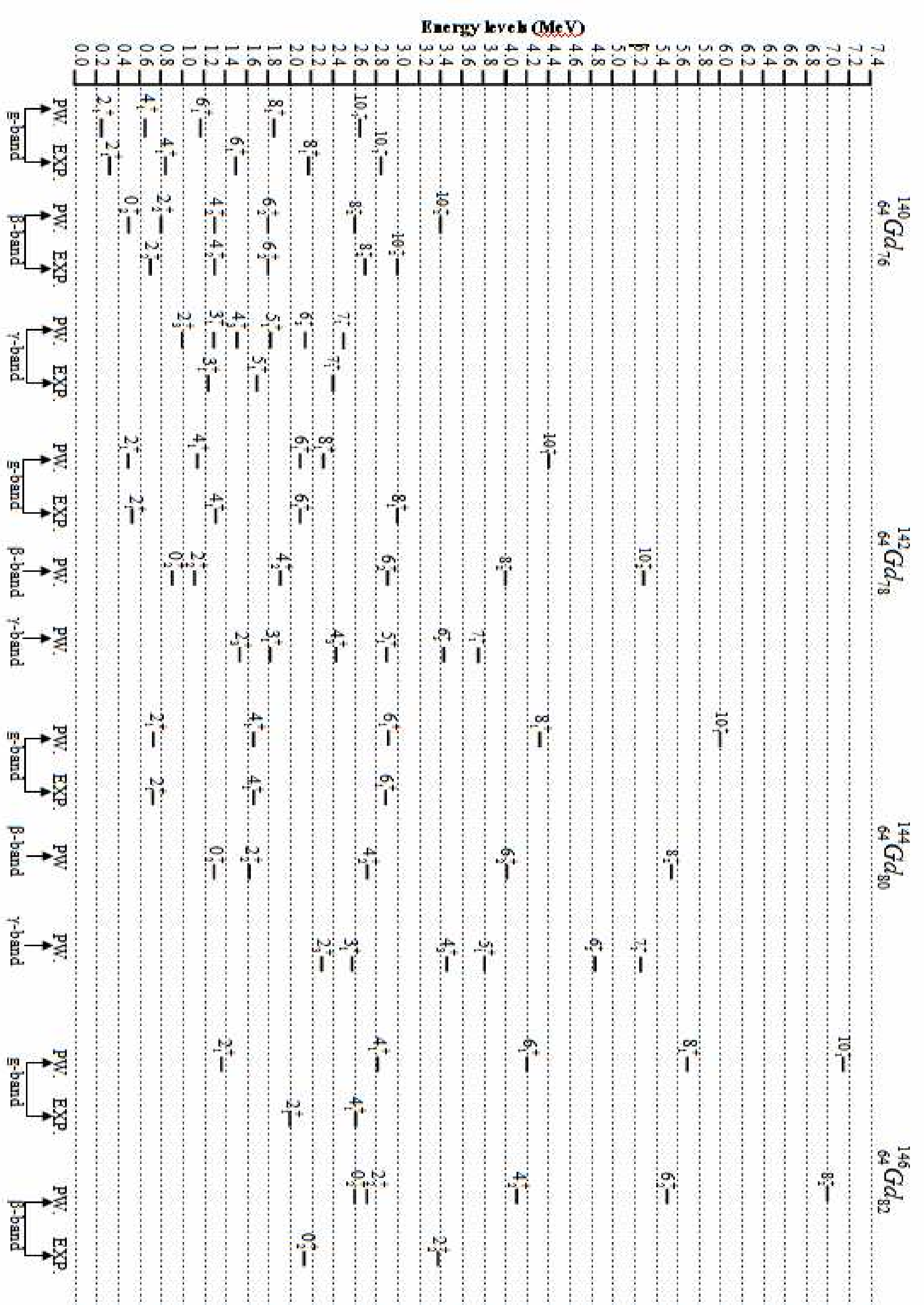

Figure (4): The energy levels of g, β, γ-bands (PW) and their comparison with the experimental bands for Gd(140-146) isotopes.

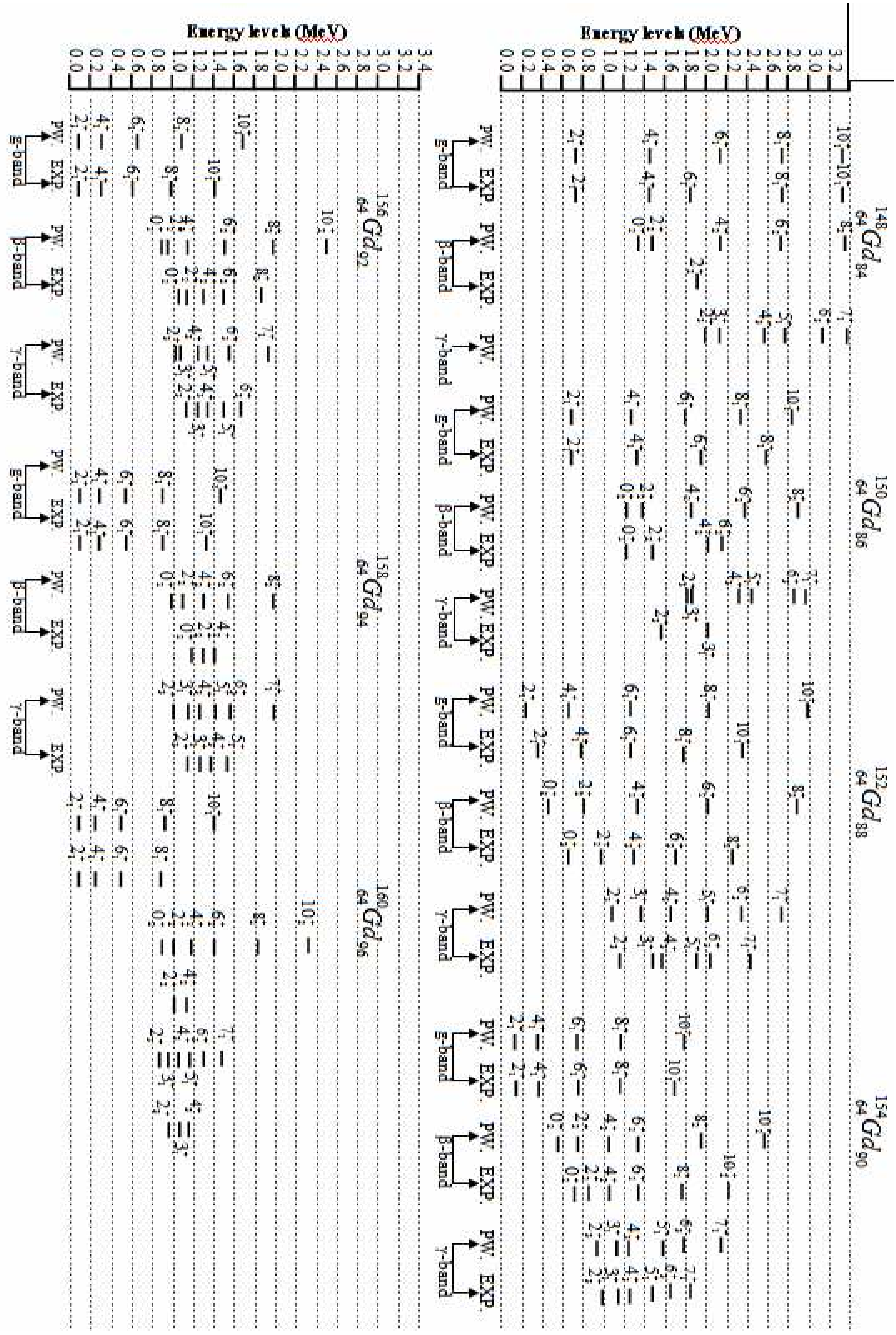

Figure (5): The energy levels of g, β, γ-bands (PW) and their comparison with the experimental bands for Gd(148-160) isotopes.

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