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Investigation Measurement of Dispersion Coefficient and Mixing Times in Bubble Column

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ABSTRACT - A complete model of liquid-phase dispersion was used to simultaneously characterize axial and radial mixing in bubble column of 0.15 m inside diameter and plastic plate distributor with holes of 2 mm diameter. Axial and radial dispersion coefficients and mixing times were determined in tap water for superficial gas velocities in the range 0.6-5.36 cm/s. The experiments were carried out using a transient method (the tracer response method). The dispersion coefficient was obtained by adjusting the experimental profiles of tracer concentration with the predictions of the model. The measured axial dispersion coefficients (Dax,L) were generally consistent with the predictions of the well established correlations, thus validating the complete dispersion model used in the analysis. The Dax,L values ranged from 110 to 200 cm²/s. There was evidence that the existing literature data on Dax,L in bubble columns are slightly underestimated, as consistent underestimation was found to be a characteristic of the widely used dispersion model that disregards radial dispersion. The value of the radial dispersion coefficient was typically about 1 to 2% of the D_{ax.L} value under any given condition. The mixing time data were generally consistent with the existing literature. The results of this study are compared with experimental result and upper bound theory and have shown a good agreement with a minimum discrepancy.

Keyword: - Bubble columns, Axial dispersion coefficient, Radial dispersion coefficient, Mixing

NOTATION

- C Tracer concentration, kmol/m³
- C_T dimensionless tracer concentration defined by Eq. (7)
- C_0 Initial concentration of the tracer, kmol/m³

- C_{∞} Final or equilibrium concentration of the tracer, kmol/m³
- C_L Liquid concentration, kg/m³
- D_{xx} Average axial turbulent eddy diffusivity according to Degaleesan et al. (1997), m²/s
- D_{rr} Average radial turbulent eddy diffusivity according to Degaleesan et al. (1997), m²/s
- $D_{ax,L}$ axial dispersion coefficient , m²/s
- D_r radial dispersion coefficient, m²/s
- D_T Column diameter, m
- g Acceleration due to gravity, m/s^2
- H Total height of the column, m
- H₀ Total liquid height in the column, m
- Hd Dispersion height, m
- J₀ zero-order Bessel function
- J₁ first-order Bessel function
- L Total liquid height in the column, m
- r Radial position, m
- R Reactor radius, m
- t Time, s
- Ug Superficial gas velocity, m/s
- UL Superficial liquid velocity, m/s
- $V_{h\infty}$ Bubble rise velocity in infinite medium, m/s
- x dimensionless radial position
- y dimensionless axial position
- z Axial coordinate, m
- ϵ_g Fractional gas hold-up
- ϵ_L Fractional liquid hold-up
- θ dimensionless time
- v_n the nth root of the first order Bessel function
- β parameter in Eq. (8)

1. INTRODUCTION

Bubble-column reactors are widely used in the chemical, petrochemical, biochemical and metallurgical industries. Their lack of moving parts and excellent heat and mass transfer characteristics are some of the prominent advantages that render them particularly attractive

for various multiphase exothermic reactions. Bubble columns are often designed with a length-to-diameter ratio, or aspect ratio, of at least 5. They are operated in either semi batch mode (zero liquid throughputs), or continuous mode (co-current or countercurrent), with liquid superficial velocities lower than the gas superficial velocity by at least an order of magnitude. As a result, it is the gas flow that controls the fluid dynamics of the individual phases in these systems. This in turn controls liquid mixing and interphase mass transfer, which subsequently influence conversion and selectivity.

of liquid phase has to be attributed to various phenomena such as turbulent vortices, liquid entrainment in the wakes of rising bubbles, large-scale liquid circulation, radial exchange flows. All these are obviously interrelated and are primarily dependent on bubble size and rise velocity distribution, gas hold-up profiles, bubble-bubble interactions and liquid circulation flow ⁽¹⁾.

Since the axial dispersion model characterises the backmixing by only a single parameter, its simplicity made it the most widely used representation of the non-ideal mixing behaviour for each phase in bubble column reactors. The time variation of the liquid phase concentration of a tracer is given by Fick's law:

Where the axial dispersion coefficient $(D_{ax,L})$ is characterized as a unique parameter for the degree of backmixing during process. The term 'axial' is used in order to mark clearly the difference between the mixing in the direction of flow and the mixing in the lateral or radial direction $^{(1)}$.

Axial dispersion coefficients of the liquid phase in vertical gas-liquid contactors have been reviewed by Shah et al., (1978)⁽²⁾. Most of reported empirical correlations indicate the dispersion coefficient to be dependent on the gas velocity and column diameter.

Ichikawa $(1967)^{(3)}$ and Chen $(1989)^{(4)}$ found insignificant effect of superficial liquid velocity on $D_{ax,L}$, whereas other authors such as Schugerl et al., $(1977)^{(5)}$ and Palaskar et al., $(2000)^{(6)}$ describe much more significant effect of superficial liquid velocity.

Baird et al., (1975) ⁽⁷⁾ modeled the most useful relation for reactor scale-up purposes and vertical cylindrical vessels with pure liquids without coalescence inhibitors. They proposed the following equation:

The dependency of $D_{ax,L}$ on D_T and U_g as expressed by Eq. (2) has been confirmed by many studies and can be also transcript in dimensionless terms of Pe and Fr numbers as represented in the following Eq.

Where constant k have different values for different authors.

As mentioned previously, the radial dispersion in bubble columns has generally been lumped with the axial dispersion coefficient and the latter has been used widely and almost exclusively as an index of mixing in such reactors. While substantial information exists on axial dispersion of fluid in Bubble columns (Ohki and Inoue, 1970⁽⁸⁾; Towell and Ackerman, 1972⁽⁹⁾. Deckwer et al., 1974⁽¹⁰⁾; Field and Davidson, 1980⁽¹¹⁾. Shah et al., 1982)⁽¹²⁾, radial mixing in these reactors has been ignored almost completely. The few measurements of radial dispersion coefficients cited by Deckwer (1992)⁽¹³⁾, suggest that the radial dispersion coefficient is always less than one-tenth of the value of the axial coefficient.

In 1996 Degaleesan⁽¹⁴⁾ developed a phenomenological model measured the axial and radial dispersion coefficients for liquid mixing, mainly based on the monitored flow circulation and turbulent eddy diffusion. The model was called "recirculation and cross flow with dispersion" (RCFD).

The model was further developed and extended also for bubble columns slurry reactors by Degaleesan et al., $(1997)^{(15)}$. The existing databases for the model were limited to airwater systems, certain column sizes and superficial gas velocities. The methodology aimed to extrapolate the data giving the following cross sectional averaged eddy diffusivities $\overline{D_{xx}}$ and $\overline{D_{rr}}$:

$$\overline{D}_{xx} = -\frac{0.00584}{D_T^{0.8}} + 0.1689 \left(D_T U_g \right)^{0.3} - - (4)$$

$$\overline{D}_{rr} = -\frac{0.000879}{D_T^{0.8}} + 0.0206 \left(D_T U_g \right)^{0.3} - - (5)$$

for $U_g \ge 0.05$ m/s.

The authors limited the applicability of the above correlations to air-water systems in churn turbulent regime only at atmospheric pressure.

The time course of a tracer's concentration at some measurement location in a bubble column can be described by the following complete dispersion mode ⁽¹⁶⁾.

Equation (6) can be made dimensionless using the following definitions of the variables:

$$\theta = \frac{D_z t}{L^2} , \qquad y = \frac{z}{L} , \qquad x = \frac{r}{L} \sqrt{\frac{D_z}{D_r}} ,$$
$$C_T = \frac{C - C_0}{C_\infty - C_0}$$

The dimensionless form of Eq. (6) is ⁽¹⁶⁾.</sup>

$$\frac{\partial C_T}{\partial \theta} = \frac{\partial^2 C_T}{\partial y^2} - \frac{U_L L}{\varepsilon_L D_z} \frac{\partial C_T}{\partial y} + \frac{1}{x} \frac{\partial C_T}{\partial x} + \frac{\partial^2 C_T}{\partial x^2} - \dots$$
(7)

Equation (7) has been solved analytically to describe complete dispersion model for batch of liquid in a bubble column⁽¹⁶⁾.

$$C_{T} = \sum_{n=1}^{\infty} \frac{J_{0}(\nu_{n}x)}{J_{0}^{2}(\nu_{n}\beta)} e^{\left(-\nu_{n}^{2}\theta\right)} \times \left(1 + 2\sum_{m=1}^{\infty} \cos(m\pi y) e^{\left(-m^{2}\pi^{2}\theta\right)}\right) \quad -- (8)$$

A similar two dimensional dispersion model has been used to characterize mixing in liquid-solid fluidized $beds^{(17,18)}$.

When the C_T in Eq. (8) is radially invariant (i.e. $D_r = \infty$), v_n, β and x become zero and $J_0 t(v_n \beta) = J_0(v_n x) = 1$. In this case Eq. (8) reduces to

$$C_{T} = 1 + 2 \sum_{m=1}^{\infty} \cos(m\pi y) e^{(-m^{2}\pi^{2}\theta)}$$
 (9)

Which is identical to the solution of the axial dispersion model that was reported by Ohki and Inoue $(1970)^{(8)}$ without considering the radial component of the dispersion.

The mixing time is defined as time necessary to achieve the homogeneity in the column, after all amount of tracer was completely mixed with the liquid.

Our objectives are to calculate the axial and radial dispersion coefficients and mixing time in bubble column from experimentally measured tracer response data obtained at superficial gas velocities in the range 0.6 - 5.36 cm/s.

2. EXPERIMENTAL

The experiments were carried out in batch type bubble column with internal diameters of 15 cm and 175 cm in height. The column was made of PVC incorporated with glass

window for the purpose of visual inspection. The column was open at the top; hence the pressure corresponded with ambient conditions. Perforated plate sparger was used in column to distribute the gas phase. The distributor plate was made of plastic plate with holes of 2 mm diameter. Air was used as the gas phase and tap water as liquid phase. The gas was introduced at the bottom of the columns. The experiments were carried out at various gas velocities, carefully adjusted and controlled using a calibrated rotameters. Before starting acquisition of data for a given gas flow rate, the system was given time to achieve steady state. A typical experimental set-up is shown in Fig. (1).

Residence time distribution (RTD) of the liquid phase was measured using different amounts of saturated solution of NaCl as a tracer. Different volumes of tracer were used to obtain the optimal amount of tracer that corresponds to optimal signal within the operating range of conductivity cell. This optimal amount of a saturated solution of NaCl was found equal to 3.38 wt %.

The conductivity probes used in this work was manufactured by Philips Company, dimensions 1cm in diameter and 15 cm long. They simply consist of two electrodes, approximately 3 mm apart, and encapsulated in plastic tubing. The probes were properly calibrated by measuring their responses to solutions of different known tracer concentrations.

The signals from the electrodes were transmitted to conductance meter (of Philips type), of range 100 μ s to 1000 ms which provide a reading in units of conductance. The meters were connected with an interface to a personal computer.

Tracer was injected as a pulse input. Local changes in tracer concentration were displayed and saved continuously on PC. Three electric conductivity probes were placed at specified radial locations (i.e. r/R = 0, 0.4 and 0.75), located at different heights as shown in Fig. (2), each of them was connected to PC via interface circuit. The distance from the injection to the measuring points, L₁, L₂, L₃ and Hd are given in Table (1).

Time for each experiment was chosen large enough in order to reach final concentration in the column.

Fig. (3) shows typical transient tracer concentrations from the column, operated at 4.68 cm/s superficial gas velocity. These signals were fitted using the analytic solution to the diffusion equation (9).

3. RESULTS AND DISCUSSION

The conductivity data used for calculation of dispersion coefficient and mixing time are fitted to be smoothed in order to remove the noise that already present due to occasional gas bubble being in contact with conductivity probes.

The dispersion coefficients are obtained by fitting the experimental data with theoretical predictions of Eq. (9). One example for fitting the measured tracer response is given in Fig. (3). clearly, the model simulates the measured data closely. The value of the radial dispersion coefficient influenced the height of the model generated peak, whereas the value of the axial dispersion coefficient influenced the width of the peak.

As shown in Fig.(4), the axial dispersion coefficient increased with increasing superficial gas velocity irrespective of the measurement location r/R. Liquid-phase turbulence, induced mainly by the movement of bubbles and the existence of large-scale liquid internal circulation, are the main causes of liquid mixing in bubble columns.

The axial dispersion coefficient values measured at the center of the column (i.e. at r/R = 0) were generally higher than the values measured at other radial locations (Fig. 4). This was because the local liquid velocity had its maximum value at the center of the column, as evidenced by the numerous measured velocity profiles in the literature ^[1, 19, and 20]. Based on these profiles, the bubble columns contain a core zone of liquid upflow (i.e. at $0 \le r/R \le 0.65$). At an approximate r/R value of > 0.7, the liquid flow changes direction.

Fig. (5) shows the measured $D_{ax,L}$ data obtained by using models with and without radial dispersion (i.e. Eqs. (8) and (9), respectively), plotted according to the form of Eq. (2). Clearly, the $D_{ax,L}$ data that does not consider radial dispersion, is quite consistent with the form of Eq. (2). The $D_{ax,L}$ values computed with the model that took into account radial dispersion (i.e. Eq. (8)) are also consistent with the behavior of Eq. (2), but are consistently slightly higher than the values computed with the model that did not consider radial dispersion (i.e. Eq. (9)). This suggests that all existing data on axial dispersion coefficients in bubble columns is a slight underestimate as all this data were determined without accounting for radial dispersion.

Fig.(6) shows the radial dispersion coefficient (Dr) values as a function of the superficial gas velocity. The data shown were obtained at 3 radial locations. Dr increased slightly as the superficial gas velocity. Further increase in superficial gas velocity did not increase the Dr value substantially. For otherwise equal conditions, the Dr value was typically only about 1 to 2% of the $D_{ax,L}$ value. The relatively low Dr values were apparently

a reflection of the fact that the bubble column operated in the bubble flow regime that exists typically when the superficial gas velocity is ≤ 5 cm/s ⁽¹³⁾. Operation in the churn turbulent flow regime that is characterized by the presence of many spherical cap bubbles, is likely to enhance the Dr value relative to the values shown in Fig.(6). Earlier work in bubble columns have documented the existence of circulating cells of gas and liquid phases in them ^[13,21]. The liquid circulation velocity in these cells is made of axial and radial components. Joshi and Sharma (1979)⁽²¹⁾ showed that the radial component of the velocity, i.e. the component that is relevant to radial mixing, is only about 36% of the axial component (in churn turbulent flow regime). This explains, at least partly, the relatively poor radial mixing in bubble columns compared to the axial mixing in them.

The measured mixing time are compared in Fig. (7) with available literatures data at same operating conditions for the columns. Fig. (7) shows that the mixing time decrease with increase in U_g. The average liquid circulation velocity increases with an increase in Ug due to increase in radial dispersion. Besides U_g, there might be an effect of the quantum of energy that is dissipated in liquid motion. The equation proposed by Joshi (1980)⁽²²⁾ for the estimation of the liquid circulation velocity contains the term $(U_g - \varepsilon_g V_{b\infty})$, which is equivalent to the quantum of energy that is supplied or available for the liquid motion. This quantity alters due to the variation in ε_g and $V_{b\infty}$, relative to the variation in Ug. It is simply stratified that when liquid circulation velocity is lower, the mixing time is higher. In other words, one can say that due to the relatively lower liquid circulation velocity values, lower energy is available for the liquid motion, which causes an increase in mixing time.

4. CONCLUSIONS

The main results presented in this work are:

- The models traditionally used for describing dispersion in bubble columns take into account only the axial dispersion. This work provides a method for simultaneously quantifying axial and radial dispersion coefficients.
- The axial dispersion coefficient values determined using the complete dispersion model was generally consistent with the predictions of the existing correlations; however, there was evidence that a disregard of radial dispersion caused a slight but consistent underestimation of the axial dispersion coefficient.

• In bubble flow regime, the value of the radial dispersion coefficient was typically about 1 to 2% of the value of the axial dispersion coefficient under given conditions of operation.

The mixing time experimental data were satisfactory with available literatures.

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Fig. (1) :Typical experimental set-up for the 15 cm diameter column.



Fig. (2): Distances to the measuring points in the column.



Fig. (3): Normalized liquid- phase tracer concentration measured at three different locations along the height of the column in response to pulse tracer injection. The smooth curves represent the fits to the curves from fitting a complete dispersion model.



Fig. (4): Effect of radial measurement position (r/R) and superficial gas velocity on axial dispersion coefficient: (a) r/R=0; (b) r/R=0.4; (c) r/R=0.75.



Fig. (5): Comparison of the Dax,L values estimated using the models with radial dispersion Eq. (8) and without radial dispersion Eq. (9), with equations of the form of Eq. (2).



Fig. (6): Effect of radial measurement position (r/R) and superficial gas velocity on radial dispersion coefficient: (a) r/R=0; (b) r/R=0.4; (c) r/R=0.75.

(b)



Fig. (7): Mixing time variation with superficial gas velocity.

Table (1): Constructional detail about the mixing experiments

Operating condition	
Liquid height H_0 /cm	135
Distance to the measuring	$L_1 = 3.5$
point / cm	$L_2 = 55$
	L ₃ =100

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قياسات بحثية لمعامل التشتت وزمن الخلط في العمود الفقاعي

الخلاصة

تم استخدام نموذج كامل لتشتت الطور السائل لوصف كل من الخلط المحوري والشعاعي في آن واحد لعمود فقاعي ذو قطر داخلي ٥١٠٠ م مع لوح بلاستيكي ذو ثقوب بقطر ٢ ملم كموزع غاز . تم قياس معاملات التشنت المحوري (Axial) والشعاعي (Radial) وزمن الخلط (Mixing time) بماء الاستهلاك المنزلي بسرع غاز بين ٢٠.- المحوري (Axial) والشعاعي (Radial) وزمن الخلط (Mixing time) بماء الاستهلاك المنزلي بسرع غاز بين ٢٠.- م مرات مراتا . نفذت التجارب باستخدام طريقة الانتقال (Mixing time) بماء الاستهلاك المنزلي بسرع غاز بين ٢٠.- مرما . نفذت التجارب باستخدام طريقة الانتقال (Transient method) اي طريقة استجابة الدليل (response experiments مراتا . نفذت التجارب باستخدام طريقة الانتقال (D_{ax.L}) اي طريقة استجابة الدليل (profile experimental) اي طريقة المخبرية (D_{ax.L}) مراتا . في تركيز الدليل مع توقعات النموذج وكانت قيم معاملات التشتت عن طريق ضبط الأشكال المختبرية (D_{ax.L}) مع نتائج العلاقات المعروفة في الأدبيات المنثورة ويذلك أكدت نجاح نموذج التشتت المصورية المقاسة ($D_{ax.L}$) مين المقاسة ($D_{ax.L}$) مع نتائج العلاقات المعروفة في الأدبيات المنثورة ويذلك أكدت نجاح نموذج التشتت المستخدم في التحليل. تراوحت قيم معاملات التشتت المحورية المقاسة ($D_{ax.L}$) مع نتائج العلاقات المعروفة في الأدبيات المنثورة ويذلك أكدت نجاح نموذج التشتت المستخدم في التحليل. تراوحت قيم معاملات التشتت الموجودة في الأدبيات المنثورة لمعامل التشتت مع نتائج العلاقات المعروفة في الأدبيات المنثورة ويذلك أكدت نجاح نموذج التشت المستخدم بشكل واسع في الأدبيات المنثورة لا يأذذ بنظر الاعتبار التشتت ألشعاعي. حيث كانت قيم معامل التشتت المستخدم بشكل واسع في الأدبيات المنثورة لا يأذذ بنظر الاعتبار التشتت ألشعاعي. حيث كانت قيم معامل التشتت المستخدم بشكل واسع في الأدبيات المنثورة لا أكمد نواح المورة لا يأذذ بنظر معامل التشت