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Numerical Analysis of Natural Frequency for U-Shaped Expansion Bellows in Fixed-Fixed and Fixed-Free Conditions

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ABSTRACT

In this study, MATLAB code was used to analyze the natural frequency in two types of U-shape metal expansion bellows with various supporting conditions (fixed-fixed and fixed- free). The first bellow has a 10 mm inner diameter., and a length of (100, 200, 300) mm, has (36, 75, and 111) convolutions. While the second bellow, with an inner diameter of 20mm and (100,200,300) mm, has a number of convolutions (25,54 and 82). The result shows that by lowering the bellow length and increasing the bellow diameter, the natural frequency was raised. Were. Where, the maximum frequency was recorded at a maximum value of 20 mm in diameter and a length of 100 mm, about (13549.167 Hz) in the case of fixed-fixed and (13097.528 Hz) in the case of fixed-free. In general, the natural frequency depends directly on the stiffness and total mass of the pipe conveying fluid. The stiffness of the material is determined by the moment of inertia. As a result, increasing the diameter will increase the moment of inertia, which will result in an increase in stiffness, which will result in an increase in natural frequency.

1. Introduction

Bellows are corrugated tubes with a thin wall that are very flexible when subjected to a variety of stress situations. [1]. Bellows are constructed of material with a rather narrow gauge (normally stainless steel), They are convoluted to provide the necessary flexibility to absorb mechanical and thermal vibrations that are expected during operation. In pipe systems, bellows expansion joints are often used to absorb differential thermal expansion while maintaining system pressure and flow under control. They are found in oil refineries., chemical plants, conventional heating and cooling systems for both home and commercial usage, as well as nuclear power plants [2]. Gawande et al. [3] examined dynamic features of U-shaped convolution bellows such as natural frequencies, modal frequency responses, and mode shapes, as well as the influence of increasing the number of convolutions on the dynamic qualities. The number of convolutions of the bellows should be increased to reduce the natural frequency for the given flow rate and avoid resonance. Lee and Chung [4] investigated the vibration in a straight pipe carrying fluid while both ends are stationary using a novel non-linear model The linearized equation is used to get the natural frequency, while the generalized-time integration approach is used to calculate the time of displacement. The validity of the new modelling is provided by comparing results from the proposed non-linear equations with those from the equations proposed by Padoussis. Veerapandi et al. [5] Using mathematical models and computer analyses, researchers investigated and examined the natural frequency in fluid-conveying pipes. An angle-type valve was discovered to be the

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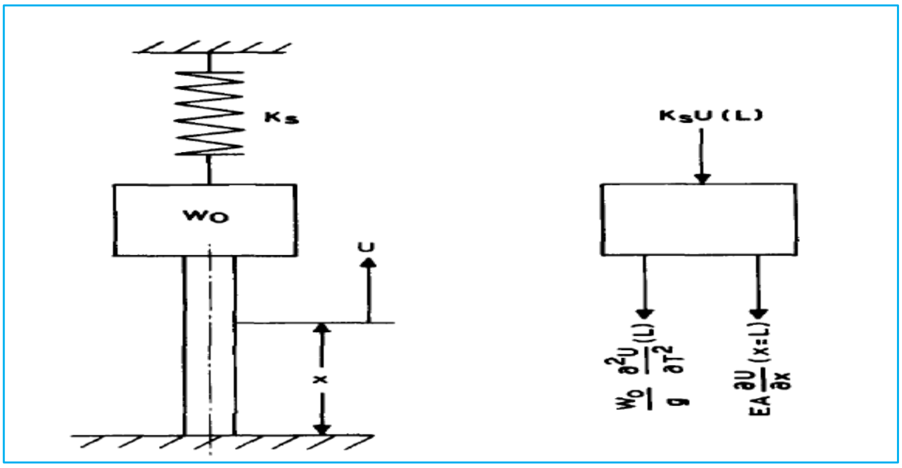
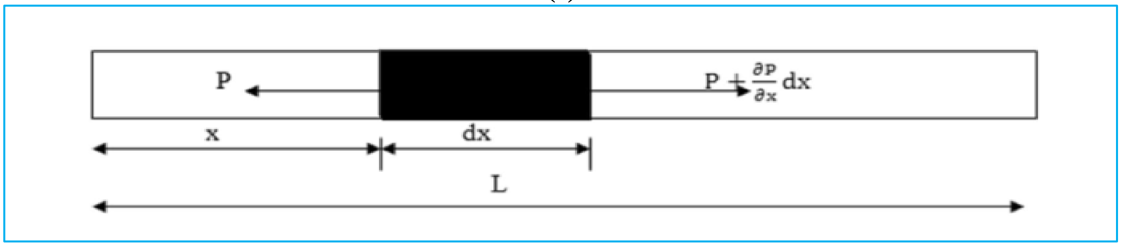
principal pipe part producing flow-induced vibration. The dynamic behavior of fluid flow via this valve is investigated analytically and quantitatively. Mao Qing et al. [6] studied the structure's inherent frequencies and responsiveness to variations in pressure and Pipeline flow acceleration as a function of flow conditions and orifice ratios. It was investigated that the test results also confirm the weakly coupled fluid-structure system assumption for the test pipe system. Avinash B. Kokare et al. [7] presented a finite element method for estimating the velocity and pressure of straight steel pipes in respect to natural frequency When the boundary conditions are fixed-free, the natural frequencies drop with increasing pipe thickness. In the case of fixed-fixed boundary conditions, however, the natural frequencies grow with increasing pipe thickness. Huang Yi-min et al. [8] Used the eliminated element-Galerkin approach, we estimated the natural frequency with various boundary conditions based on the usual transverse vibration mode. The link between the natural frequency of the pipeline transporting fluid and that of the Euler beam has been investigated, it is concluded. Finally, a dimensionless flow velocity and limit values are supplied, which may be utilized to determine the flow velocity's influence on the natural frequency. Naguleswaran and Williams [9] For hinged-hinged, fixed-hinged, and fixed-fixed boundary conditions, Natural frequency solutions in axial mode were investigated. The precise solution is compared to approximate one and two term Gaierkin, Rayleigh-Ritz, and Fourier series solutions. A two-term Galerkin or Rayleigh-Ritz solution is found to be straightforward to analyze and offers a decent approximation of frequency and phase. Becquet et al. [10] For their work, they investigated innovative closed-form solutions for the natural frequency of a fixed-guided beam. Using the clamped–guided beam's mode shape and material density and stiffness as polynomial functions, one may derive the closed-form solution for its natural frequency. The proposed method's great simplicity is a unique feature. Seo et al. [11] Used a finite element technique, researchers investigated the frequency analysis of cylindrical shells and determined the fluid's effective thickness. circumferential modes in the study. The finite element approach was used to create a cylindrical shell transporting fluid with uniform velocity. A beam-like shell element is used instead of a conventional shell element. Furthermore, This strategy might require fewer elements than the traditional shell-type elements. The accuracy of this method was not inferior to that of conventional shell-type elements. Chellapilla and Simha [12] The findings for three basic boundary conditions, pinned-pinned, pinned-clamped, and clamped-clamped, were computed using Fourier series and Galerkin techniques. Results are shown for varied values of both foundation stiffness parameters, and a comparison with known literature is performed for the situation of the second parameter being equal to zero, as well as novel results for varying values of the second foundation parameter. Interesting findings are reached about the influence of foundation factors on pipeline critical flow velocity. Jweeg et al. [13] The critical buckling velocities for pinned–pinned, fixed–pinned, and fixed–fixed end conditions were determined by tests, and the basic natural frequencies for the end conditions were measured. Experiments on two pipe models with three distinct boundary conditions were carried out. According to the results, the current method is more accurate for determining the critical velocities of pinned-pinned and clamp-pinned pipes. However, this was not the case for clamp-clamp pipes until a larger flow rate was applied. Huang et al. [14] A mathematical model based on the Ferraris technique of natural frequency analysis was created for a pipeline delivering fluid with both ends supported. The natural frequencies and critical flow velocities for fixed-simply supported, fixed-fixed support, and simply-simply supported instances were also identified, with the fixed-fixed example considered to be more stable than the other two due to the higher overall stiffness of the system. The primary goal of this article is to calculate the natural frequency of U-shaped bellows in two distinct types of supports.

2. Mathematical model

In this study, the bellows is modeled as a long continuous rod or a long pipe in dynamic analysis, with an elemental mass (m) and an elemental stiffness (Ks). The solution of the general governing equation for the natural frequency is derived using Timoshenko beam theory. Additional bellows with various end boundary designs. The first is the bellows, which has one fixed end and two free ends. The second component is a bellows with both ends fastened. Consider the length (L) of a beam or straight pipe with a constant area of cross-section (A) depicted in Figure (1-b). Consider the little elemental length (dx) at a distance (x) from the pipe's one end. The axial load applies at (x+dx) in the opposite direction of (p).



(a)



(b)

Figure 1. a- free body diagram (FBD)of bellows [15] b- Longitudinal beam [15]

$P+\frac{\partial P}{\partial x}$

Axial strain in rods, $u = \frac{PL}{AE}$

Therefore, at the elemental length, the axial strain is expressed as, $\partial u = \frac{Pdx}{AE}$

But, $P = AE\frac{\partial u}{\partial x}$ and $\frac{\partial P}{\partial x} = AE\frac{\partial^2 u}{\partial x^2}$, The equation may be used to calculate the axial natural frequencies of bellow using beam theory.

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} \qquad (1)$$

Where (u) is the pipe's axial displacement in millimeters and (T) is the duration in seconds.

$$a=\sqrt{E/\rho} = \sqrt{Eg/v}$$

where (E) is the pipe material's young's modulus in Mpa, (g) is the acceleration due to gravity in mm/s^2, and (v) is the pipe's weight per unit volume in N/mm^3.

The general solution for (1) is;

$$u(x,t) = \sum_{i=1}^{\infty}[A_i \sin\left(\frac{\omega}{a}\right)x + B_i \cos\left(\frac{\omega}{a}\right)x][C_i sin\omega t + D_i cos\omega t\ ] \qquad (2)$$

Assuming that one end of the pipe is stationary, i.e., x = 0, and displacement u (0, t) = 0.

Using the aforementioned boundary condition in eq. (2), we get;

$$u(0,t) = [o + B_i][C_i sin\omega t + D_i cos\omega t] = 0$$

$$\rightarrow B_i[C_i sin\omega t + D_i cos\omega t] = 0$$

Here $[C_i sin\omega t + D_i cos\omega t] \neq 0$
$B_{i=0}$
By substituting value in equation (2), we obtain;

$u = \sum_{i=1}^{\infty}[A_i \sin\left(\frac{\omega}{a}\right)x][C_i sin\omega t + D_i cos\omega t]$ (3)

It is now assumed that a straight pipe with continuously and evenly distributed mass is fixed at its bottom in order to create a mathematical model of bellows. A weight W_0 is affixed to its top and is connected by a spring. Now, using the D'Alembertz principle, the equilibrium equation may be expressed as follows:

$$\mathrm{AE}\ \frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \frac{\mathrm{W_0}}{\mathrm{g}}\frac{\partial^2 \mathrm{u}}{\partial \mathrm{T}^2} + \mathrm{K_s u} = 0 \quad (4)$$

where A is the cross-sectional area of the bellow in mm2, and we obtain by differentiating equation (3) with regard to x and t;

$$\frac{du}{dx} = [\frac{A_i}{a}\omega cos\left(\frac{\omega}{a}\right)x][C_i sin\omega t + D_i\ cos\ \omega t].$$

$$\frac{du}{dt} = [A_i \sin\left(\frac{\omega}{a}\right)x]\ [\ C_i \omega cos\omega t - D_i\ \omega sin\ \omega t]$$

$$\frac{d^2u}{dt^2} = [-A_i sin\left(\frac{\omega}{a}\right)x][-C_i\omega^2 sin\omega t - D_i\ \omega^2 cos\ \omega t]$$

$$\frac{d^2u}{dt^2} = -\omega^2[A_i sin\left(\frac{\omega}{a}\right)x][C_i sin\omega t + D_i\ cos\ \omega t]$$

We obtain by substituting the aforementioned values in equation (4)

$$AE[A_i\frac{\omega}{a}\cos\left(\frac{\omega}{a}\right)x][C_i sin\omega t + D_i\ cos\ \omega t] - \frac{\mathrm{W_0}}{\mathrm{g}}\ \omega^2[A_i sin\left(\frac{\omega}{a}\right)x][C_i\ sin\ \omega t + D_i\ cos\omega t] + K_s\ [\ A_i sin\left(\frac{\omega}{a}\right)x][\ C_i sin\omega t + D_i\ cos\ \omega t]\ = 0$$

With x=L
We get even more simplicity.;

$$\mathrm{AE}\ A_i\frac{\omega_\mathrm{i}}{\mathrm{a}}\cos\frac{\omega_\mathrm{i}}{\mathrm{a}}\mathrm{L} - \frac{\mathrm{W_0}}{\mathrm{g}}\omega_\mathrm{i}^2\ A_i \sin\frac{\omega_\mathrm{i}}{\mathrm{a}}\mathrm{L} + \mathrm{K_s}\ A_i \sin\frac{\omega_\mathrm{i}}{\mathrm{a}}\mathrm{L} = 0 \quad (5)$$

The term L/a may be simplified further as;

$$L/a = \sqrt{(LAv\ /(AEg\ /\ L)} = \sqrt{(G/g)}/(P/(L\ \partial u/\partial x)} = \sqrt{(G/(g\ Kn\ )} \quad (6)$$

where (L) is the straight pipe length, a=$\sqrt{Eg}/_v$,G is the pipe weight (N), P is the applied axial force (N), and Kn is the pipe's axial spring constant (N/mm).

$G = LAv$

$$\mathrm{E} = \frac{\mathrm{P}/_\mathrm{A}}{(\frac{\partial \mathrm{u}}{\partial \mathrm{x}})} \quad \text{and } K_n = \frac{P}{L(\frac{\partial u}{\partial x})} \quad , \mathrm{P} = AE\ \frac{\partial \mathrm{u}}{\partial \mathrm{x}}$$

By varying the value of equation (6) in equation (5), we obtain:

$$AE\frac{\omega_i}{a}\cos\left(\omega_i\sqrt{\frac{G}{gK_n}}\right) - \frac{W_0}{g}\omega_i{}^2\sin\left(\omega_i\sqrt{\frac{G}{gK_n}}\right) + K_s\sin\left(\omega_i\sqrt{\frac{G}{gK_n}}\right)\ = 0 \quad (7)$$

The fundamental natural frequencies of the bellows may be calculated using the preceding equation under various end circumstances.

Condition 1- The bellows has one fixed end and one free end.

In this case, the conditions means that the $\mathrm{W_0} = 0$ and $\mathrm{K_s} = 0$, $\mathrm{W_0}$(It represents the weight of element that we took and it is very little, so we consider its value to be= 0).

When we substitute the values in equation (7), we get:

$$\cos\omega_\mathrm{i}\sqrt{(\mathrm{G}/(\mathrm{gKn}\ )}\ = 0 \quad \rightarrow cos\omega_\mathrm{i}\sqrt{\frac{\mathrm{G}}{\mathrm{gK_n}}} = 0 \text{ taking } \cos^{-1} for\ both\ sides$$

$$\rightarrow \omega_i\sqrt{\frac{\mathrm{G}}{\mathrm{gK_n}}} = cos^{-1}(0)$$

where: *i* the order number of natural frequency is known as, it is the counter of angles, which represent the roots of the solution (i = 1, 2, 3, 4….)

$$\mathrm{f_i} = \frac{\omega_\mathrm{i}}{2\pi}$$

$$\rightarrow 2\pi\ \mathrm{f_i} = \sqrt{\frac{\mathrm{gK_n}}{G}}\ (\mathrm{i} - 0.5)$$

$$\rightarrow f_{i=}\sqrt{\frac{K_n}{G}}\ .\frac{\sqrt{g}}{2\pi}.(i - 0.5)\ \pi$$

The frequency equation may be written as follows by inserting g = 9806.65mm/s2:

$$f_i = 49.5\ (i - 0.5)\sqrt{\frac{K_n}{G}} \quad (8)$$

Condition 2- The bellows are attached on both ends.

In this case, the conditions means that the $W_0 = 0$ and $K_s = \infty$,W_0(It represents the weight of element that we took and it is very little, so we consider its value to be= 0).

When we substitute the values in equation (7), we get:

$$\sin\omega_i \sqrt{(G/(gK_n)} = 0 \quad \rightarrow \quad \omega_i \sqrt{(\frac{G}{gK_n}} = sin^{-1}(0)$$

$$\rightarrow \omega_i \sqrt{\frac{G}{gK_n}} = i\pi \quad \rightarrow \quad f_i = \sqrt{\frac{K_n}{G}} \cdot \frac{\sqrt{g}}{2} \cdot i$$

$$f_i = 49.5\ i \sqrt{\frac{K_n}{G}} \quad (9)$$

Table 1: bellow expansion specification

Parameter	Bellow (1)	Bellow (2)
Expansion joint material	SS 304 L	SS 304 L
Modulus of elasticity	193000 N/mm^2	193000 N/mm^2
Bellow Yield stress	215 N/mm^2	215 N/mm^2
Poisson's ratio	0.275	0.275
Bellow type	U-Shaped	U-Shaped
Bellows expansion length	(100, 200, 300) mm	(100, 200, 300) mm
Outside diameter	13.6	25.2 mm
Inner diameter	10 mm	20 mm
Thickness	1mm	1 mm
Depth of convolution	0.8 mm	1.6 mm
Convolution pitch	2.5 mm	3.8 mm
The total number of Convolutions	36,75,111	25,54,82
End tangent length	40 mm	15 mm

3. Results and discussion

3.1. Natural frequency analysis

A MATLAB code was used to order and numerically solve the equations of natural frequency in fixed-fixed and fixed-free conditions. Six programming codes were built to calculate and graph the natural frequency variation with order number. The first three codes deal with the expansion bellow with an inner diameter of 10 mm and with lengths (100, 200, and 300) mm and the number of convolutions (36, 75, and 111) convolutions.

The second three codes deal with the expansion bellow with an inner diameter of 20 mm and a length of(100, 200, and 300) mm and a (25, 54, and 82) number of convolutions. The construction and run of the programming code depend on the material properties of the expansion bellow (304 L) and design parameters. Modulus of elasticity, density, and poison ratio are among the qualities. While the design parameters, as stated in the table, comprise the inner diameter, number of convolutions, convolution pitch and breadth, material thickness, bellow length, and tangent diameter (1), Figure depicts the flow chart for running the MATLAB algorithms (2).

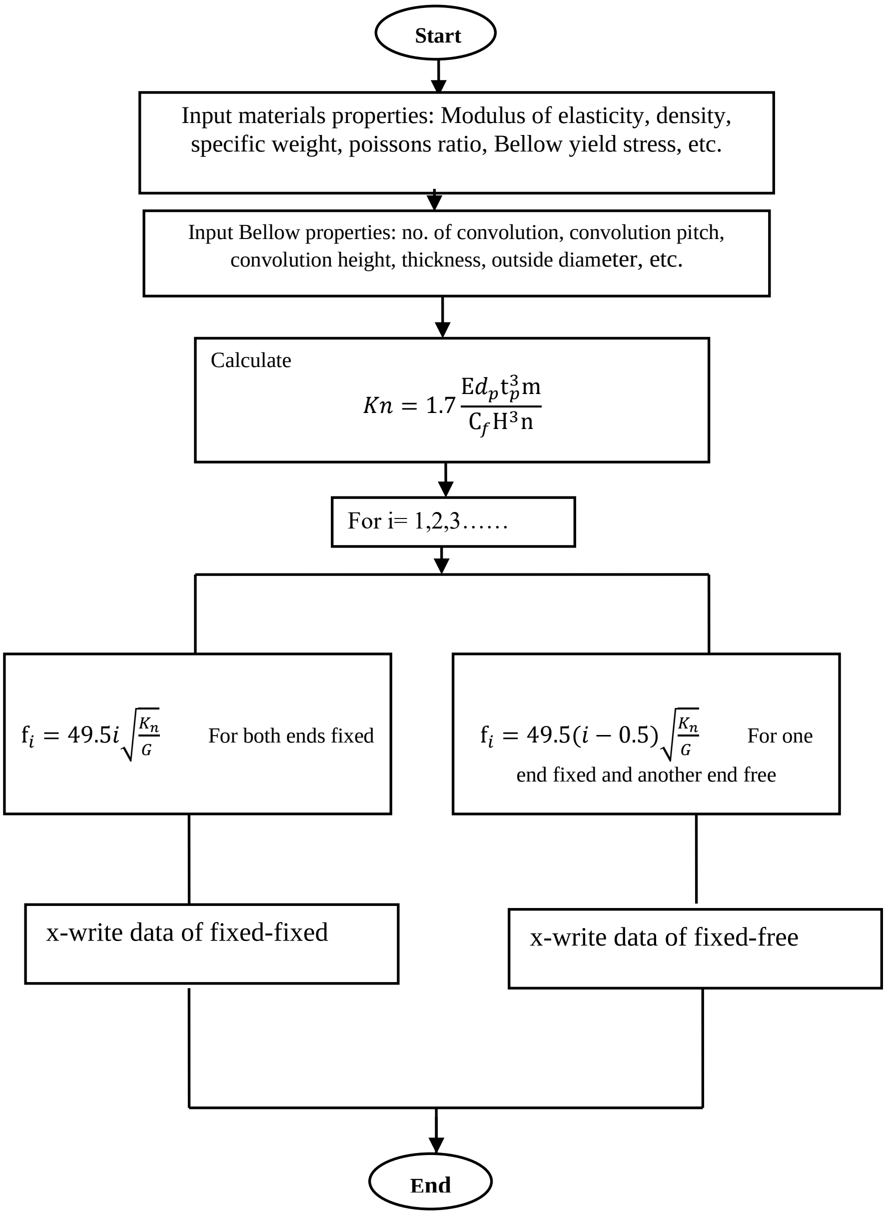
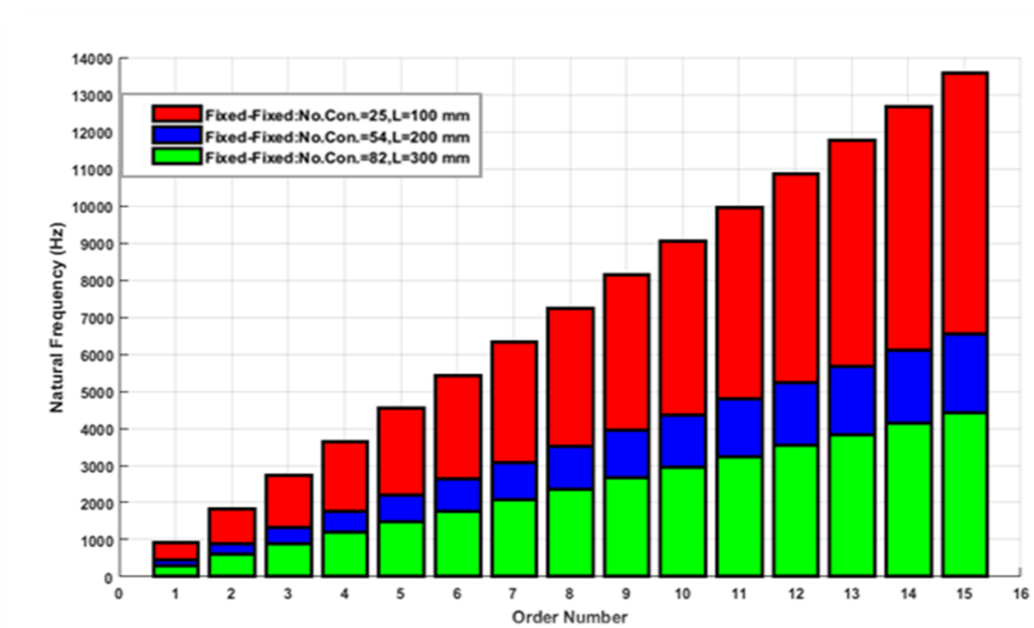


Figure 2. Flow chart of natural frequency in expansion bellow with fixed-fixed and fixed-free support

The variation of natural frequency with the order number of natural frequencies for the expansion bellow at an inner diameter of 20 mm and different lengths was presented in Figure (3). As shown in figure (3), the natural frequency decreased with increasing the number of convolutions in two examples of assistance (fixed-fixed and fixed-free). In the instance of fixed-fixed, the natural frequencies were in the range of (903.277 and 13549.167 Hz) for the number of convolutions n = 25, (435.8475 to 6537.713 Hz) for n = 54, and (293.99059 to 4409.8588 Hz) for n =82, as shown in Table (2). In addition, the natural frequencies in the case of fixed-free ranged between (451.63890 to 13097.528 Hz) for the number of convolutions n = 25, (217.92378 to 6319.7899 Hz) for n = 54, and (146.99529 to 4262.8635 Hz) for n =82.

In other words, the proportion of decreasing natural frequency as the number of convolutions increases. in the case of fixed-fixed support was recorded at 51.7% when the number of convolutions increased from 25 to 54, while the decreasing percentage was recorded at 67.4% when the convolutions of the expansion bellows were increased to 82. In the same manner, the percentage of decreasing natural frequency was recorded with the same behavior and values as in the case of fixed-free support.



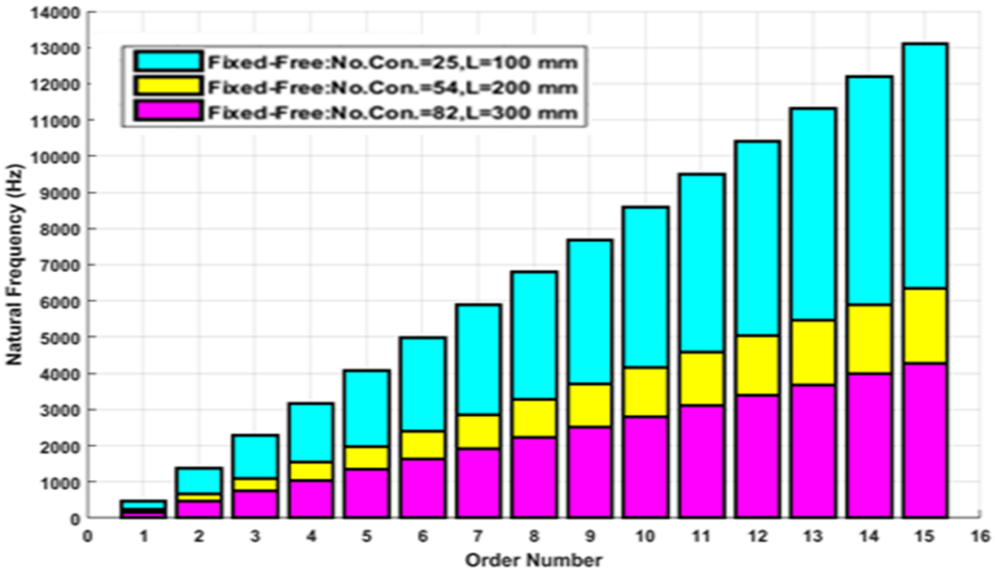


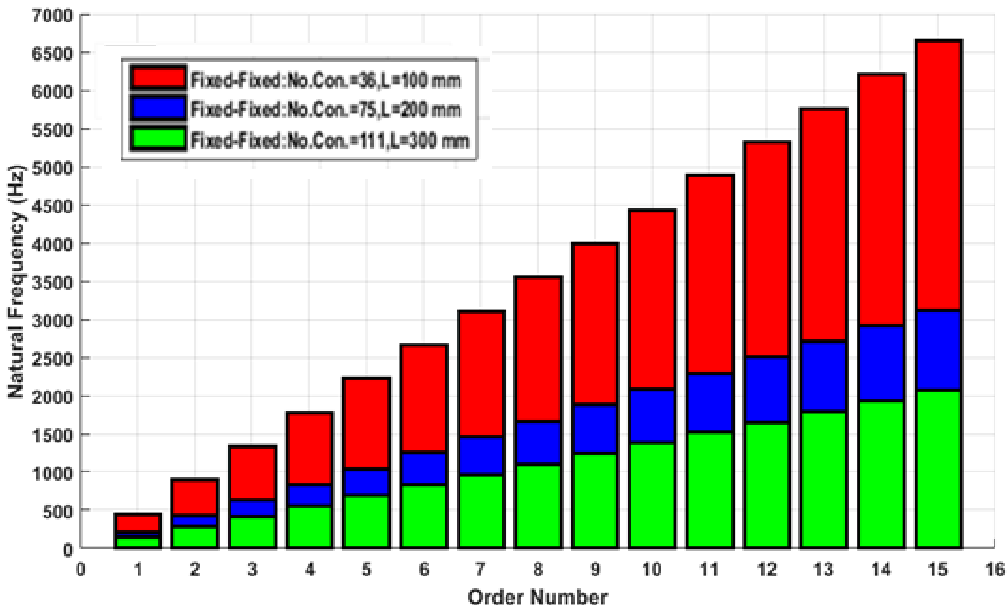
Figure 3. Natural frequency variation with number of order (inner diameter 20 mm)

Table 2: Values of natural frequencies in cases of fixed-fixed and fixed –free supports for 20 mm inner diameter

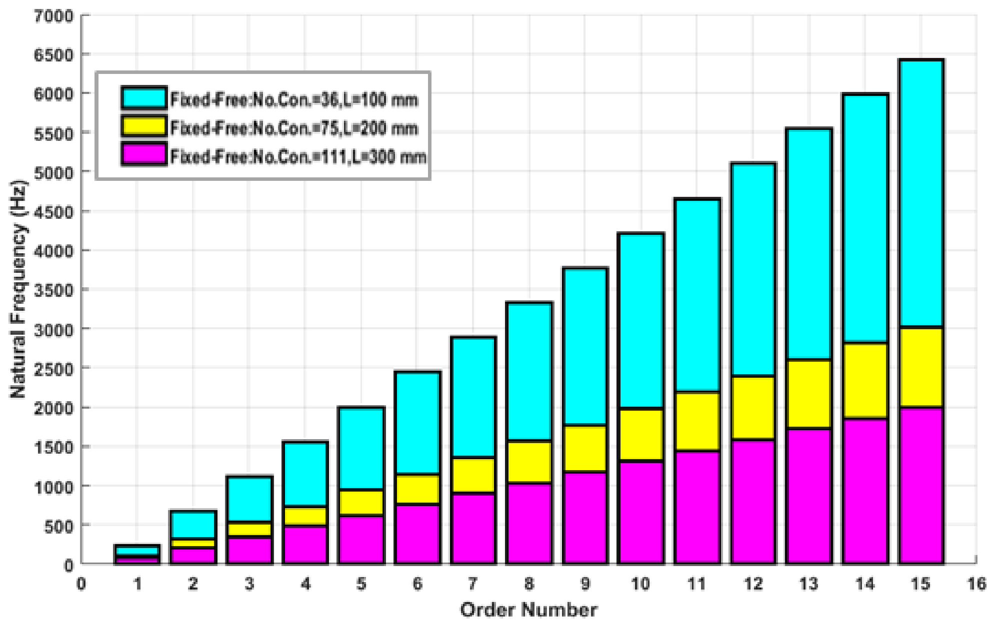
Order number	Natural Frequencies (Hz)					
	Fixed-fixed			Fixed-free		
	L=100, No.=25	L=200, No.=54	L=300, No.=82	L=100, No.=25	L=200, No.=54	L=300, No.=82
1	903.277	435.8475	293.99059	451.63890	217.92378	146.99529
2	1806.555	871.6951	587.98118	1354.9167	653.77136	440.98588
3	2709.833	1307.542	881.97177	2258.1945	1089.6189	734.97647

4	3613.111	1743.390	1175.9623	3161.4723	1525.4665	1028.9670
5	4516.389	2179.237	1469.9529	4064.7501	1961.3141	1322.9576
6	5419.666	2615.085	1763.9435	4968.0279	2397.1616	1616.9482
7	6322.944	3050.933	2057.9341	5871.3057	2833.0092	1910.9388
8	7226.222	3486.780	2351.9247	6774.5835	3268.8568	2204.9294
9	8129.500	3922.628	2645.9153	7677.8613	3704.7044	2498.9200
10	9032.778	4358.475	2939.9059	8581.1391	4140.5520	2792.9106
11	9936.055	4794.323	3233.8964	9484.4169	4576.3995	3086.9012
12	10839.3337	5230.170	3527.8870	10387.694	5012.2471	3380.8917
13	11742.611	5666.018	3821.8776	11290.972	5448.0947	3674.8823
14	12645.889	6101.866	4115.8682	12194.250	5883.9423	3968.8729
15	13549.167	6537.713	4409.8588	13097.528	6319.7899	4262.8635

The natural frequency of an expansion bellow with an inner diameter of 10 mm has the same behavior as the natural frequency with an inner diameter of 20 mm. As shown in Figure (4), In two support examples, the natural frequency declined as the number of convolutions increased (fixed-fixed and fixed-free). In three scenarios of the number of convolutions, the natural frequency was higher in the case of fixed-fixed support than in the case of fixed-free support. As observed in Table (3), in the instance of fixed-fixed, the natural frequencies varied between (442.8204 to 6642.3307 Hz) for the number of convolutions n = 36, (207.76507 to 3116.4760 Hz) for n = 75, and (137.10392 to 2056.5589 Hz) for n = 111.



(a)



(b)

Figure 4. Natural frequency variation with number of order (inner diameter 10 mm)

In addition, the natural frequencies in the case of fixed-free ranged between (221.41102 to 6420.9197 Hz) for the number of convolutions n = 36, (103.88253 to 3012.5935 Hz) for n = 75, and (68.551964 to 1988.0069 Hz) for n = 111. In other words, the proportion of decreasing natural frequency as the number of convolutions increases. in the case of fixed-fixed support was recorded at 53.08% when the number of convolutions increased from 36 to 75, while the decreasing percentage was recorded at 69.03% when the convolutions of the expansion bellows were increased to 111. In the same manner, the percentage of decreasing natural frequency showed the same behavior and values in the case of fixed-free support.

Table 4: Comparative of natural frequencies with previous studies

References		**Material characteristics and design parameters**						**Type of support**	**Natural frequency Hz**
	Material type	**Metal thickness mm**	**Bellow type**	**Outside diameter mm**	**No. convolution**	**Convolution pitch mm**	**Convolution height mm**		
A.B.Kadam et al.	SA-240 321	0.9	U	147	6	8	8	Fixed-Fixed	1792.89
								Fixed-Free	574.58
					7			Fixed-Fixed	1729.86
								Fixed-Free	503.21
Vishnu Rajan and L.G. Navale	SA-240 321	0.9	U	147	10	8	8	Fixed-Fixed	1713.1
								Fixed-Free	578.60

Present study	304 L	0.8	U	13.6	36	2.5	0.8	Fixed-Fixed	442.822
					75				207.7651
					111				137.104
				25.2	25	3.8	1.6		903.277
					54				435.848
					82				293.9906
				13.6	36	2.5	0.8	Fixed-Free	221.4110
					75				103.8825
					111				68.55196
				25.2	25	3.8	1.6		451.6389
					54				217.9238
					82				146.9953

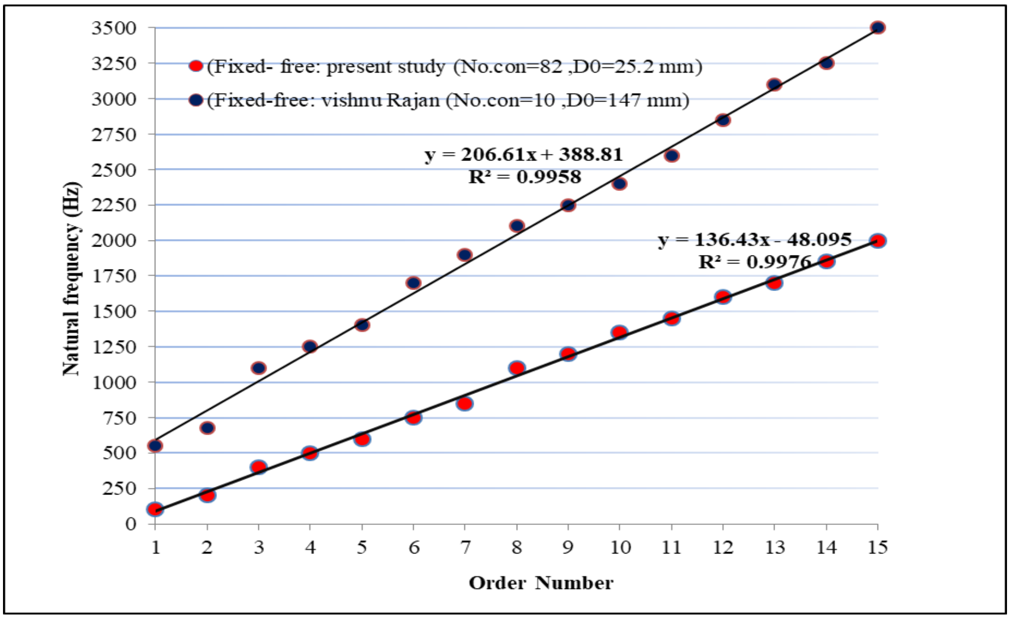


Figure 5. Comparison between the natural frequency of the present study with the previous study

4. Conclusions

A MATLAB code was used to solve the natural frequency in fixed-fixed and fixed-free conditions for six scenarios of metal expansion bellows. The first three codes deal with expansion bellows with an inner diameter of 10 mm and with lengths and number of convolutions (100, 200, and 300) mm and (36, 75, and 111) respectively. The second three codes deal with expansion bellows with an inner diameter of 20 mm, a length of (100, 200, and 300) mm, and a number of convolutions of (25, 54, and 82). The following conclusions may be reached from the examination of code results:

1. As the bellows length is decreased, the natural frequency increases.
2. The natural frequency increases with increasing the bellow diameter.
3. The values of natural frequencies in fixed-fixed support were higher than in fixed-free support, because the overall stiffness of the system was higher.
4. The natural frequency increases with decreasing the number of convolutions.

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Notations and Nomenclatures

symbols	*Description*	*Unit*
A	Cross sectional area	mm^2
A_i,B_i, C_i , D_i	Constant	
dx	Element length	mm
E	Elastic modulus of the pipe material	Mpa
f_i	Frequency	Hz
G	weight of the pipe	N
g	Gravitational acceleration	9806.65 mm/s^2)
i	The order number of natural frequencies, i = 1, 2,3,4…	
K_n	The pipe's axial spring rate	N/mm
L	Length of straight pipe	mm
P	Applied force	N
T	Time	s
u	Axial displacement of the pipe	mm
v	pipe material weight per unit volume	N/mm^3
W0	weight of element connected	kg
x	Distance from one end of the pipe	mm
	Greek symbol	
ω	The natural frequencies of bellow	Hz