Dynamic load factor in finite cracked bodies under harmonic loading

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Abstract:
A method of computing dynamic load factor under harmonic load is suggested. The method is based on the representation of the dynamic stress intensity factors in form corresponding to normalized free vibration modes with some weight coefficients and then determine dynamic load factor of 2024-T3 alloy. The motion equation is solved by ANSYS program to determine the effect of harmonic loading on dynamic load factor and fracture toughness from another side.

Introduction:

The numerical methods of liner fracture mechanics are now developed enough for solving a wide range of static problems [1]. In recent years ,attempts were made to apply these methods for determination of the stress intensity factors under dynamic impact loading [2]. However, it is often necessary to taken into account the inertia effect caused by harmonic loading, as neglecting of this effect leads to unreliable value of failure load [3]. It occasionally that the structural members, which are safe under static load, break down when dynamic load act on them.

In present research will be determined the dynamic stress intensity factors in the finite plates with two types of cracks, central sharp crack and central circular crack under axial harmonic extension-compression .as well as in the other known papers dealing with the determination of dynamic stress intensity factors, the interaction of crack surfaces has not been taken into account.

Mechanical properties:

Mechanical properties of 2024-T3 alloy.

Yield strength, MN/m^2 (σ_y) 250

Modulus of elasticity, (E) G N/m^2 72.34

Poisson's ratio (ν) 0.33

Ultimate tensile strength (σ_{Ts}), Mpa 325

Strain hardening exponent (n), 0.2

Shear modulus (G), G N/m^2 , 27

Elongation %, (ΔL) 15mm

Fracture stress, (σ_F) Mpa 170

Reduction area, (RA%) 13

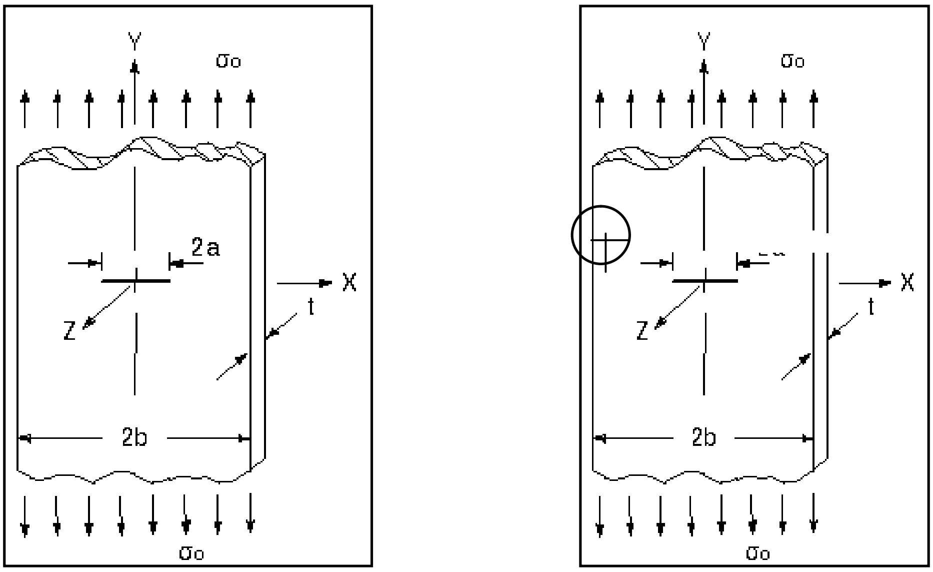
Harmonic Analysis:

This problem is also of theoretical interest as the known analytical dependencies of stress intensity factors on frequency in finite cracked bodies other words dynamic stress intensity factor $\left(k_{I(FEM)}^{dyn}\right)$ of paper's specimens .The research's result depends on two types of specimens with different cracks as follows (15mm, 25mm). To solve present problem will be deal with finite element method by Ansys program to determine the dynamic data. The dynamic load factor has been found from taken the ratio between the dynamic stress intensity factor and static stress intensity factor (the last is limited from practical test). Dynamic test is consisting of applying axial harmonic extension – compression with maximum load that had been determined from practical test (tensile test). The max. Load of two types of specimens are (135Mpa, 120 Mpa) for specimens with central circular cracks and (131.5 Mpa, 111.8Mpa) for specimens with central sharp cracks. Also, the load of unit intensity was applied on the horizontal edge .The whole model of two type of specimens are shown in figure (1).

Because of important the field near the crack tip from side of stress values and distributed method, it's data will be depended in building the research's result and then find the dynamic load factor (DLF) as end step, which it is represented the ratio between the dynamic stress intensity factor $\left(k_{I_{(FEM)}}^{dyn}\right)$and the static stress intensity factor $\left(k_I^{static}\right)$[1]. The finite element equation of motion of an elastic body under harmonic loading is:

$$M\ddot{x} + kx = fe^{i\omega t} \text{......(1)}$$

Where M is the mass matrix, k is the stiffness matrix, x is displacement vector, and f is loading vector. This equation is solving by Ansys program.



a) Specimen with central sharp crack b) specimen with central circular crack

Fig. (1) whole model of two type of specimens

Determine the vibration phases

Vibration phase of specimens must be found at first. This step is very important in harmonic analysis, and then becomes ability to limit the natural frequency of each specimen. For example we will illustrate the important steps of natural frequency limitation of specimen with sharp crack (15mm). These steps will treatment by Ansys program. The first step must be fixed this specimen from one end to prevent any side motion and by depend on all model available in Ansys program, will be determine the correct phase that related with specimen behavior as illustrated in the following table. The same manner will be depended with other specimens.

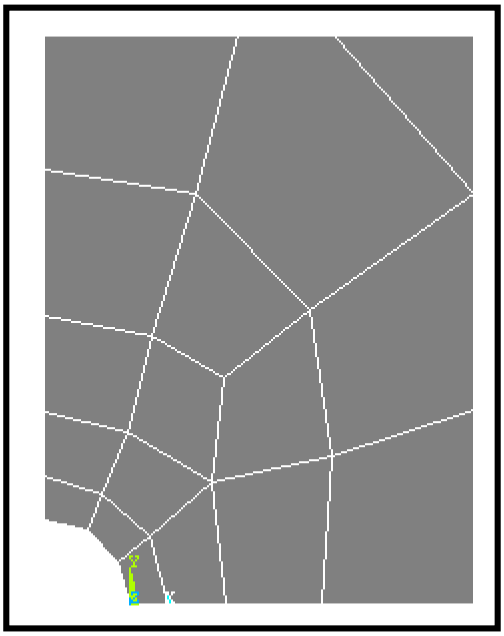
Table (1) natural frequency of all types of research's specimen with different cracks length and vibration phase.

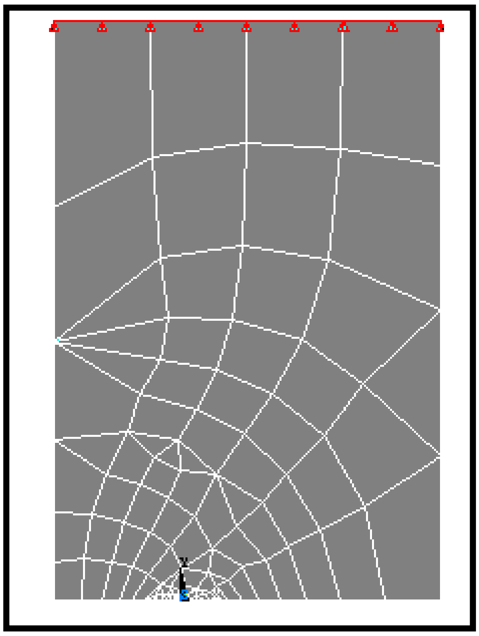
The Specimens type	Crack length (mm)	Phase No.	natural frequency
Central sharp crack	15mm	1	17
	25mm	1	19
Central circular crack	15mm	1	18
	25mm	1	23

The main aims from limitation natural frequency of each specimen is to determine the maximum stress under harmonic loading and relate this parameters with frequency in relationship .The frequency range is between (0-25 Hz). This procedure is applied to know the effect of this loading on stress data at crack tip and to know how it is distributed around the crack tip zone. The finite element grid is showing in fig. (2).

Two- dimensional dynamic stress intensity factor computation using Ansys program

In two – dimensional computation for linear elastic fracture mechanics, how to simulate the stress singularity near the crack tip has been difficult and important point The so-called quarter – point element is often to used to model the stress field near the crack tip [4]. However, Ansys only provides automatic meshing capability for two – dimensional problem .At the crack tip region we generate the quarter – point element manually to model the correct singularity of stress near the crack tip, thus making the computation for two – dimensional crack problem possible in this paper, one method are represented to computed two – dimensional dynamic stress intensity factor $\left(k_{I_{(FEM)}}^{dyn}\right)$. Manual generation is only needed for sub model region, which is much reduced size; thus manual generation is feasible. Secondly, mesh [5] element can be used to mesh the area with two-





dimensional singular element, This method is easy to handle and extend the ability of Ansys in computing two- dimensional $\left(k_{I_{(FEM)}}^{dyn}\right)$.

(a) (b)

Fig. (2) The mesh of quarter model (quarter specimen under harmonic loading)

a) Specimen with central circular crack. b) Specimen with central sharp crack.

<u>Analysis of dynamic stress intensity factor</u>

The harmonic response of specimens under frequency range (0-25 Hz) it's clear from the relationship between the equivalent stress (σ_{equ}^{dyn}) at crack tip and harmonic frequency as shown in figures (3) and (4), from these figures we note that the maximum stress of specimens with central sharp crack and central circular crack is happening when the entrance is occurred between the natural and forced frequency. The numerical results of the dynamic and static stress intensity factor are shown in table (2) for two types of specimens.

Table (2) dynamic stress intensity factor of two types of specimen under harmonic loading.

Specimen type	Crack Length (mm)	Equivalent static stress at crack tip (σ_{eq}^{static}) *Mpa*	Equivalent dynam stress at crack tip (σ_{eq}^{dyn}) *Mpa*	$(k_{I_{(FEM)}}^{dyn})$ $\text{Mpa}\sqrt{\text{m}}$	(k_I^{static}) $\text{Mpa}\sqrt{\text{m}}$
Central sharp crack	15 25	167 172	176 195	28 41	26.26 36.29
Central circular crack	15 25	186 230	691 868	153 168	37 44.518

The effect of harmonic loading upon the equivalent stress at crack tip region will be clearance from fig. (5), consider the area of equivalent stress around crack tip zone, will be cleared that the equivalent stress area are graduated distribution from maximum value (read color) to minimum value. Also, the same thing is happened with other crack case.

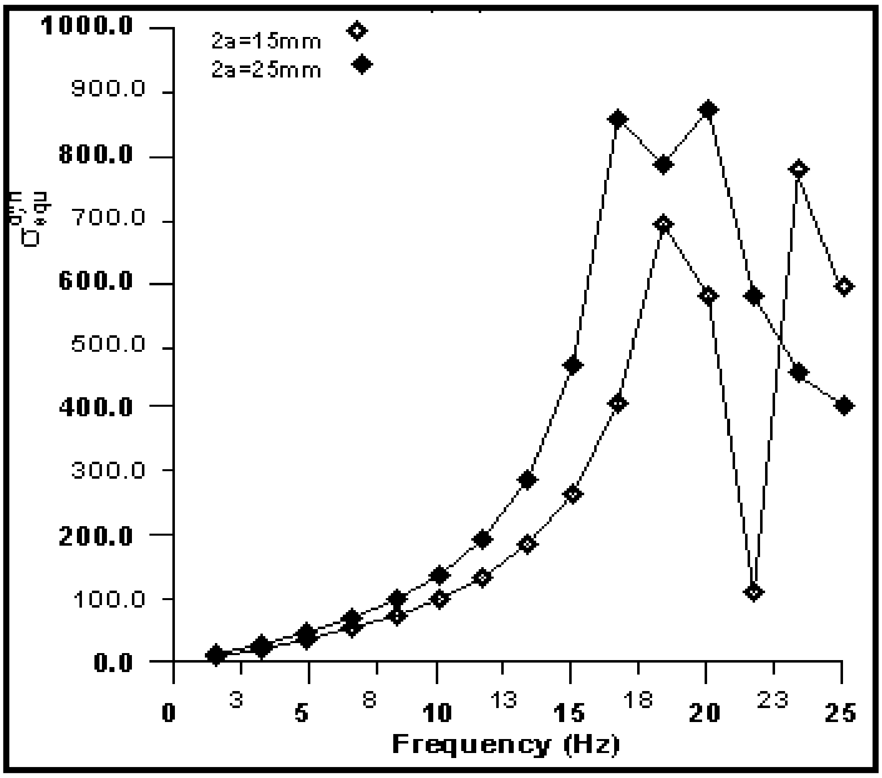


Fig. (3) harmonic response of specimen with central circular length (25 mm) under frequency range (0-25Hz).

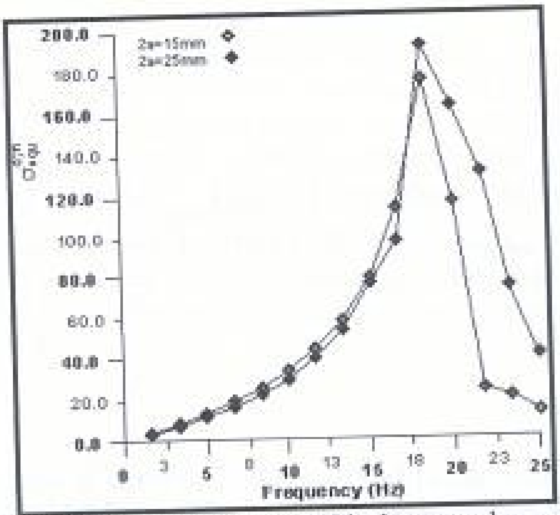


Fig. (4) harmonic response of specimen with sharp crack crack length under frequency range (0-25Hz).

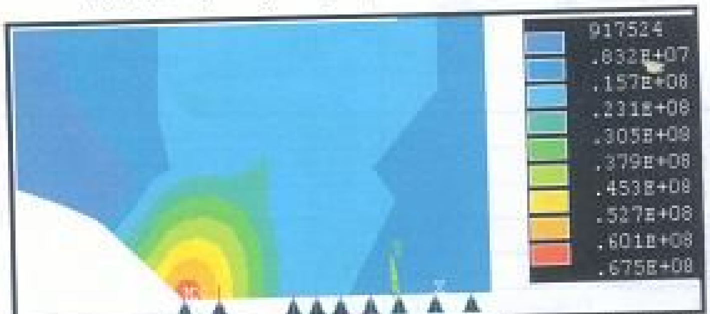


Fig. (5) equivalent stress around crack tip zone of specimen with central sharp crack length under frequency range (0-25Hz).

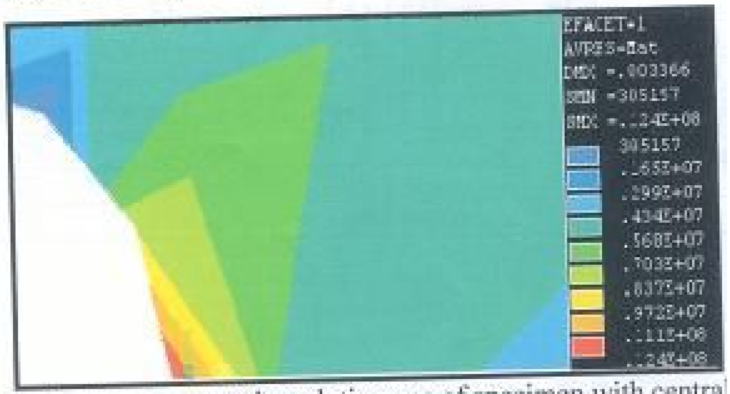


Fig. (6) equivalent stress around crack tip zone of specimen with central circular crack length (25mm)

Determine the dynamic load factor (DLF)

Without doubt the harmonic frequency has the large effect in rise the equivalent stress value in dynamic loading more than the static loading .As a result the difference between the two loading values will be happened .The different between two loading, static and dynamic can be determine from the ratio between dynamic and static stress intensity factor $\left(k_{I(FEM)}^{dyn}\right)$, $\left(k_{I}^{static}\right)$ respectively. This ratio is known as dynamic load factor and denoted by DLF. To know the harmonic frequency effecting on DLF, the numerical results have been plotted as DLF vs. frequency in figure (7) and (8) according to type of specimen and crack length.

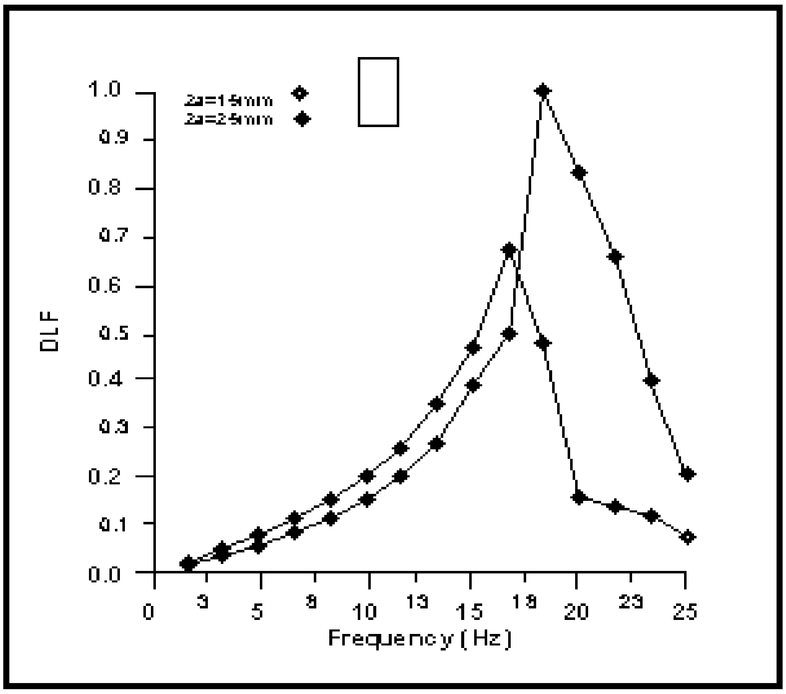


Fig. (7) harmonic frequency effecting on (DLF) of specimen with central sharp crack with different crack length.

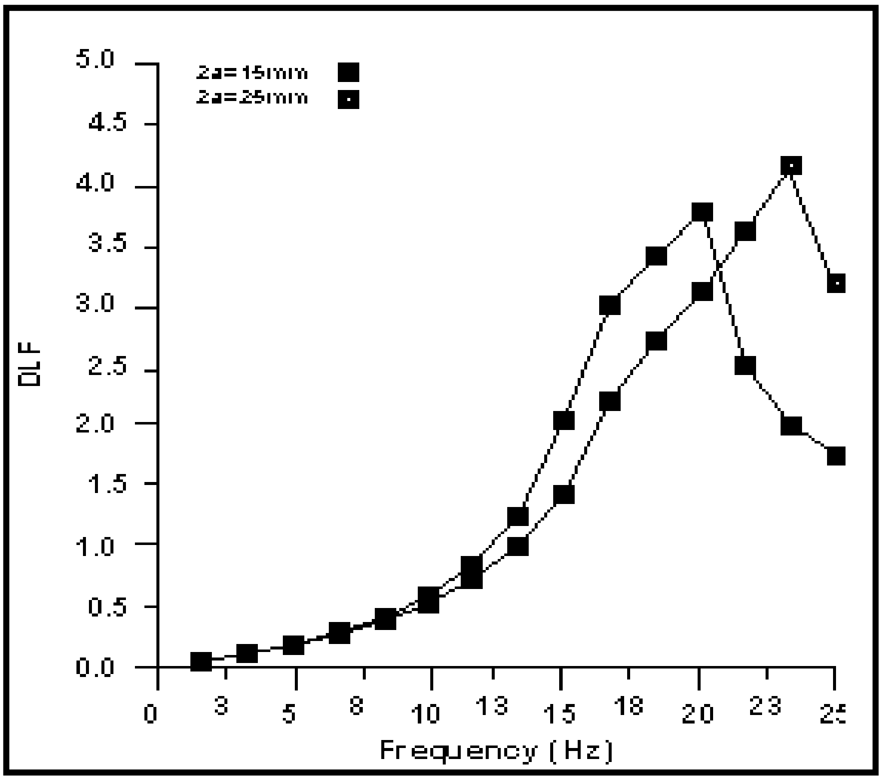


Fig. (8) harmonic frequency effecting on (DLF) of specimen with central circular crack with different crack length.

Be clear the DLF increase when the harmonic frequency is increasing also seems the relationship between the crack length and harmonic frequency is inverse as shown in figure (7)& (8) and the difference between peaks means unstable cracks growth under a loading condition. This behavior of two specimens is happening because of the entrance between natural frequency and harmonic frequency, at this time the danger of brittle failure increase under high harmonic frequency loading.

<u>Determine the dynamic Fracture toughness of 2024-T3 alloys under harmonic frequency.</u>

The dynamic fracture toughness of 2024-T3 alloy is limited from take the ratio between the dynamic stress intensity factor and yield stress $(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$ [6] illustrated in table (5) respected with two type of specimens and crack length, then plotted the relationship between the ratio harmonic frequency and $(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$.

Table (5) the result data of dynamic fracture toughness for two types of specimens.

Frequency (Hz)	$(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$ specimens with crack length (15mm)		$(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$ specimens with crack length (25mm)	
	central circular crack type	central sharp crack type	central circular crack type	central sharp crack type
1.667	0.01	0.0027	0.011	0.0031
3.333	0.02	0.0056	0.025	0.065
5	0.03	0.0087	0.038	0.01
6.667	0.045	0.0122	0.056	0.014
8.33	0.061	0.0163	0.078	0.019
10	0.081	0.0214	0.108	0.025
11.667	0.109	0.0279	0.15	0.034
13.33	0.149	0.037	0.22	0.045
15	0.212	0.0496	0.365	0.0645
16.667	0.32	0.0708	0.66	0.01
18.33	0.55	0.11	0.61	0.168
20	0.46	0.0169	0.67	0.138
21.667	0.89	0.0152	0.449	0.12
23.33	0.616	0.1308	0.356	0.064
25	0.47	0.0812	0.31	0.0345

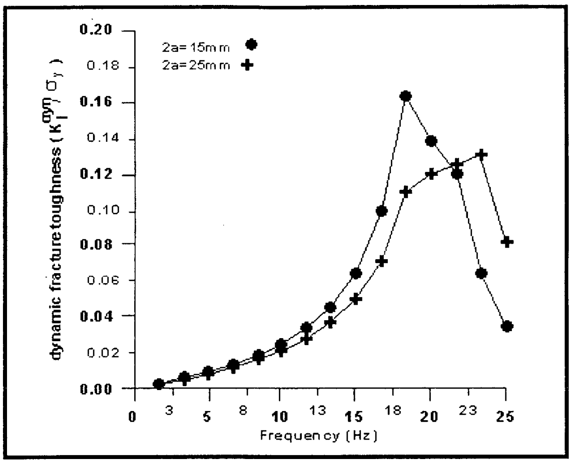


Figure (9) dynamic fracture toughness with harmonic frequency $(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$ of specimen with central sharp crack.

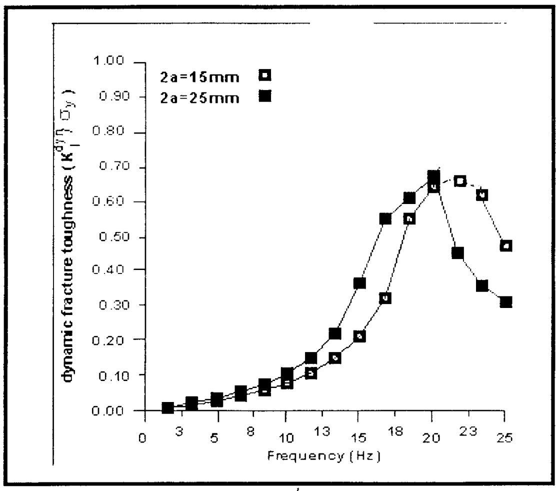


Figure (10) harmonic frequency with ratio $(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$ of specimen with central circular sharp crack.

From figure (9) and (10) we note that ratio $(\frac{k_{I(FEM)}^{dyn}}{\sigma_y})$ increase when the frequency increase. This increase will be made increasing in plastic zone at crack tip and lowing in yield toughness of research's alloy. This process includes absorbing energy insurance reach to critical energy (G_c), which lead to critical stress intensity factor. This means that the danger of brittle failure increase under harmonic high frequency loading for two types of specimens, but the largest valve is happen with specimen with central circular crack panel.

Conclusions

A method of calculation of dynamic stress intensity factor in cracked vibrating plates has been suggested. Finite element method has been employed for determine the dynamic stress intensity factor and then determine the DLF, as a result to predict fracture instability. Harmonic extension- compression loading type has been chosen. Research's specimens consist of two types different in crack type (sharp and circular) and cracks length (25mm, 15mm). Fracture toughness has been calculated at end. The main results of the research are:

(When the harmonic frequency increases)

1. The equivalent stress and then equivalent dynamic stress intensity factor around crack tip increase also.
2. Dynamic load factor DLF increase around crack tip and this process lead to large different between the data of static test for dynamic test (under harmonic loading).
3. Probability of fracture instability is increasing, when harmonic loading takes in view with static loading.

These results refer to unreliable data of failure if data depend on static analysis only. These lead to increase the danger of brittle failure under harmonic loading. Therefor, to prevent brittle fracture of mechanical structure must be taken care whether the dynamic stress intensity factors is smaller than the dynamic fracture toughness value o research's material or not.

Reference:

1) BORISSKOVSKY, V.G, et. Al " On the dynamic stress intensity factors in finite cracked bodies under harmonic loading", Eng.Fracture Mechanics vol.16, No.4, pp.459-465, 1982.
2) K.Kishimoto, S.Aoki and M.Sakata " Simple formula for dynamic stress intensity factor of pre-cracked charpy specimen". Eng. Fracture Mech.13, 501-508, (1980).
3) J.W.Kelley, C.T.Sun, A singular finite element for computing time dependent stress intensity factors. Eng. Fracture Mech.12, 13-22, (1979).
4) Jia, X.M, Dai.F. " Three – dimensional static and dynamic stress intensity factor computations using Ansys." From Internet, at three – dimension static and dynamic, (www. dynamic stress intensity factor, 2004).
5) C.W.Woo and M. D.KURUPPU, " Experimental determination of crack growth resistance of ductile materials", Journal of Strain Analysis vol.16, No.2, 1998.
6) Hahn.G.T. "Dynamic fracture toughness parameters for HY-80 and HY-130 steel and their weldments." Engineering Fracture Mechanics, vol.14, No.4, pp.725-740, 1981.