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Abstract

A new generalization of faintly continuous function was defined and some basic properties were given. Correlation between faintly simply-continuous function and simply-connected spaces was proved, the property of simply-normal and simply-compact space had been studied.

Keyword: Simply-open set, simply-continuous function, faintly simply-continuous function, simply-connected, slightly simply – continuous, simply-compact space.

الدوال المستمرة الضعيفة من النوع البسيط

سهاد كريم حميد و جلال حاتم حسين

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الخلاصة

تم تعريف تعميم جديد للدوال المستمرة الضعيفة ودراسة خصائصها ومميزاتها، وتمت اثبات العلاقة بين الدوال المستمرة الضعيفة نوع البسيط والفضاء المتصل، اضافة الى الخصائص الاخرى التي تم مناقشتها واثباتها مثل الخاصية الفضاء العادي وخاصية الفضاء المضغوط.

الكلمات المفتاحية: المجموعات المفتوحة نوع simply، الدوال المستمرة الضعيفة نوع simply، الفضاءالتبولوجي المتصل، الفضاء التبولوجي المضغوط.

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Introduction

In topology, a weak form of continuity played very important role in topology by using different forms of weak open set. In 1982, a researcher Long and Herrington [1] defined the faintly - continuous function by using ɵ-open set. T. Noiri and V. Popa [2] investigated a three weak types of faintly-continuity which are the faintly precontinuous, faintly semicontinuous,and faintly β −continuous that was in 1990. Nasef [3, 4], had introduced two forms of faintly continuous by the term of strong faintly α-continuous and strong faintly γ-continuous and he discussed the relation between the concept of strong faintly α-continuous and the other types of continuity. In this Paper, we introduce a new generalization of faintly continuous through the term of simply-open set that defined by A.Neubrunnova [5] and simply-continuous function in the sense of Biswas [6]. Moreover, some properties of faintly simply-continuous function were given.

1. Preliminaries

In this paper we will use the symbols (X and Y) to refer to topological spaces unless otherwise mentioned. Here some fundamental definitions that we need to know in the paper.

Definition 1.1: A subset A of a topological space X is called:

i. Semi-open set [7] if $A \subset Cl[Int(A)]$
ii. Pre-open set [8] if $A \subset Int[Cl(A)]$
iii. ɵ-open set if for each $x \in A$,there is an open set U such that $x \in U \subseteq clU \subseteq A$
iv. Simply-open set [4] if $A = G \cup N$, where G is an open set and N nowhere dense set [5]. Where N nowhere dense set if $\left(Cl(N)\right) = \emptyset$.

Definition. 1.2: [1] A function f:(X, τ)→(Y,σ) is said to be faintly continuous if $f^{-1}(V)$ is open in X for every ɵ-open set V of Y.

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Definition .1.3: [6] The function f:(X,τ)→(Y, σ) is called simply-continuous function at x if for each open set V of Y consisting $f(x)$, there an exists a simply-open set U with $x \in U$ *such that* $f(U) \subset V$.

Definition 1.4: The function f:(X, τ)→(Y,τ) is called faintly simply –precontinuous at a point $x \epsilon X$ if for each ɵ-open set U of Y consisting $f(x)$, there exists a simply-preopen set V such that $f(V) \subset U$.

Example 1: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, \{b, c\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$, f:(X,τ)→(Y,σ) is define by $f(a)= a$, $f(b)= b$, $f(c)= a$,then f is faintly simply-precontinuous function.

Definition 1.5: A function f:(X, τ)→(Y, σ) is said to be slightly simply – continuous if for each $x \epsilon X$ and for each clopen set U of Y consisting $f(x)$, there exists a simply-open subset V such that $f(V) \subset U$.

Example 2: Let$X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$,and let the function f:(X,τ)→(Y,σ) is the identity function ,then f is slightly simply-precontinuous function.

2. Faintly simply- continuous function

Definition 2.1: The function f:(X, τ)→(Y, σ) is called faintly simply - continuous at a point $x \epsilon X$ if for each ɵ-open set U of Y consisting $f(x)$, there exists a simply-open subset V s.t. $f(V) \subset U$. f is called faintly simply-continuous if its satisfy this property at each point of X .

Theorem 2.2: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ the following statements are equipollent:

i. f is faintly simply-continuous.

ii. $f^{-1}(V)$ is simply-open in X for each ɵ-open set V *of* Y .

iii. $f^{-1}(G)$ is simply-closed in X for each ɵ-closed G in Y.

Proof: (i)→(ii): Suppose V be ϴ-open set of Y, let $x \in f^{-1}(V)$,then $f(x) \in V$ and f is faintly simply-continuous. There exists a simply-open set U of X such that $f(U) \subset V$,therefore $x \in U \subset f^{-1}(V)$ .So we find a simply-open set U containing x and subset of $f^{-1}(V)$.Hence $f^{-1}(V)$ is simply-open in X .

(ii)→ (i): Let $x \in X$ and V be an ϴ-open subset of Y consisting $f(x)$. By (ii) $f^{-1}(V)$ is a simply-open subset containing x . Take $U = f^{-1}(V)$, then $f(U) \subset V$, therefore f is faintly simply-continuous.

(ii) →(iii): Let V be a closed set of Y ,then Y/V is ϴ-open set from (ii), we get that $f^{-1}(Y/V) = X\backslash f^{-1}(V)$ is a simply-open. Hence $f^{-1}(V)$ is simply-closed.

(iii) →(ii): Suppose that V be an ϴ-open set of Y ,then V^c is ϴ-closed in Y. By (iii) the $f^{-1}(Y/V) = X\backslash f^{-1}(V)$ which is simply-closed and therefore the $f^{-1}(V)$ is simply-open.

Theorem 2.3: Let $f:(X,\tau) \to (Y,\sigma)$ be a simply-continuous then f is a faintly simply-continuous.

Proof: Let $x \in X$,let V be ϴ-open subset of Y consisting $f(x)$. Since f is simply-continuous function at x, then, there is a simply-open set U in X containing x s.t $f(U) \subset V$. Therefore f is faintly-simply continuous.

Definition 2.4: The function $f:(X,\tau) \to (Y,\sigma)$ is called weakly simply-continuous (respectively almost simply-continuous) if $\forall\ x \in X$ and each open subset V of Y consisting $f(x)$, there is a simply-open set U of X such that $f(U) \subset Cl(V)$ (respectively $f(U) \subset Int[Cl(V)]$.

Theorem 2.5: If a function $f:(X,\tau) \to (Y,\sigma)$ is a weakly simply-continuous, then the function is faintly simply -continuous.

Proof: Let $x \in X$ and V be ϴ-open subset of Y containing $f(x)$. So, there is an open set W with $f(x) \in W \subset cl\ (W) \subset V$(from definition of ϴ-open set). Since f is weakly simply-continuous, then there exists a simply-open set U containing x with $f(U) \subset cl(W) \subset V$. Hence, f is faintly simply-continuous.

Theorem 2.6: Every faintly simply-continuous function is faintly simply-precontinuous function.

Proof: Let $x \in X$, V be a ɵ-open subset of Y consisting $f(x)$, there is a simply-open subset U of X s.t $f(U) \subset V$ (since f is faintly simply-continuous). Since U is a simply-open set then U is simply-preopen [9]. Therefore f is a faintly simply-precontinuous.

Definition 2.7: A topological space (X, τ) is called almost-regular if $\forall$ regular closed set G of X and $\forall$ point $x \notin G$, there exist a disjoint open set U , V of X with $x \notin U$ and $G \subset C$.

Theorem 2.8: For the function $f: (X, \tau) \to (Y, \sigma)$, the following statements are equivalents:

1. f is almost simply-continuous at $x \in X$
2. For every regular open subset V consisting $f(x)$, there exists a simply-open set U in X s.t. $f(U) \subset (V)$.

Proof: (1→2): Let, $x \in X$ and W be an open set of Y containing $f(x)$, since f is almost simply-continuous, then there exists a simply-open set U of X such that; $f(U) \subset Int[Cl(W)]$, W is open set then its preopen set [10] then $W \subset Int[Cl(W)]$. Hence $f(U) \subset W$.

(2→1): Let, $x \in X$, W be an open set and let V be regular open subset consisting $f(x)$ such that $V \subset W$,then there exists a simply-open set U of X such that $f(U) \subset V$. Now V is regular then, $V = Int[Cl(V)]$, that implies $f(U) \subset Int[Cl(V)]$, with $V \subset W$. Hence $f(U) \subset Int[Cl(W)]$, therefore f is almost simply-continuous at $x \in X$.

Theorem 2.9: If the function $f: (X, \tau) \to (Y, \sigma)$ is a faintly simply-continuous with (Y, σ) is an almost regular then, f is an almost simply-continuous.

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Proof: Let $x \notin X$, V be any regular open subset of Y consisting x . Since every regular open subset in an almost regular topological space is a ɵ-open [11], V is ɵ-open and we have f is faintly simply-continuous, then there exists a simply-open set U in X s.t. $f(U) \subset (V)$. From theorem (2.8) f is almost simply-continuous.

Corollary 2.10: Let (Y, σ) is almost –regular topological space, so for a function $f: (X, \tau) \rightarrow (Y, \sigma)$ the following statement are equipollent:

1. f be an almost simply – continuous
2. f be a weakly simply – continuous
3. f be a faintly simply – continuous

Theorem 2.11: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is a faintly simply – continuous and (Y, σ) is regular, so f is simply – continuous.

Proof: Suppose U be open subset of Y, and we have Y is regular by hypothesis, U is $\theta - open$ in Y .Since f is faintly simply – continuous by theorem we get $f^{-1}(U)$ is simply – open and hence f is simply continuous.

Theorem 2.12: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a faintly simply continuous. Therefore f is a slightly simply – continuous.

Proof: Let $x \in X, V$ be a clopen subset of y consisting $f(x)$. So V is $\theta - open$ in Y. We have f is faintly simply – continues then there is a simply – open set U in X containing x s.t. $f(U) \subset V$ that leads the function f is slightly simply continuous.

Definition 2.13: A space (X, τ) is called simply-connected if X cannot be represented with a disjoint union of the two nonempty simply – open subsets.

Theorem 2.14: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is faintly simply-continuous function and (X, τ) is simply-connected topological space therefore Y is connected topological space.

Proof: Claim that (Y, σ) isn't connected topological space. Therefore, there exist a nonempty open set V_1 *and* V_2 such that $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = Y$. Now we have $f^{-1}(V)_1 \cap f^{-1}(V_2) = \emptyset$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = X$ since f is surjective , $V_1 \cup V_2$ is $\theta - open$ set (since V_1 *and* V_2 is open sets then the infinite union of open set is open ,then $V_1 \cup V_2$is open and every open set is $\theta - open$), and f is faintly simply-continuous then $f^{-1}(V_1), f^{-1}(V_2)$ is simply-open that implies : (X, τ) isn't connected that is a contradiction so Y is connected space.

Definition 2.15: A space : (X, τ) is called simply-compact space if $\forall$ simply open cover of X has a finite subcover.

Theorem 2.16: The subjective faintly simply-continuous image of a simply-compact space is θ-compact.

Proof: Let $f:(X, \tau) \to (Y, \sigma)$ is faintly simply-continuous function and X is simply-compact space. Suppose $\{G_\alpha : \alpha \in I\}$ be θ-open cover of. Since f is faintly simply-continuous the collection $\{f^{-1}(G_\alpha): \alpha \in I\}$ is a simply-open cover of, X is simply-compact space, then there exist a finite sub cover $\{f^{-1}(G_i): i = 1,2, \dots, n\}$ of X . That implies $\{(G_i): i = 1,2, \dots, n\}$ is finite subfamily which cover Y .Hence Y is θ-compact space.

Separation Axioms

Definition 2.17: A topological space (X, τ) is called:

1. Simply-T_1 if for each pair of distinct point x , y of, then there exist simply-open subsets U *and* V consisting x and y respectively s.t. $y \notin U$, $x \notin V$.
2. Simply - T_2 if for each pair of distinct points x and y in, there exists disjoint simply – open sets U *and* V *in* X s.t. $X \in U$ *and* $Y \in V$.

Definition 2.18: [12] A topological space (X, τ) is said to be:

1.$\theta - T_1$ if for each pair of distinct points x, y in X, there exist θ-open subsets U *and* V consisting x and y respectively s.t. $y \notin U$, $x \notin V$.

2. $\theta - T_2$ if for each pair of distinct points x and y in X , there exists disjoint θ – open sets $U\ and\ V\ in\ X$ s.t. $x \in U\ and\ y \in V$.

Theorem 2.19: If $f: (X, \tau) \to (Y, \sigma)$ is faintly simply continuous injection and $Y\ is\ \theta - T_1$ space, then $X\ is\ a\ simply - T_1$ space.

Proof: Let $Y\ is\ a\ \theta - T_1$space. For any distinct point $X\ and\ Y\ in\ X$. There exists $\theta - open\ set\ V, W$ such that $f(X) \in V$, since $f$is faintly simply – continuous, then we get

$f^{-1}(V) and\ f^{-1}(W)$ are simply open subsets of (X, τ) such that $X \in f^{-1}(V)$, $Y \notin f^{-1}(V)$, $X \notin f^{-1}(W)$, $Y \notin f^{-1}(W)$. That implies $X\ is\ a\ simply - T_1$ space.

Theorem 2.20: If f is faintly simply – continuous injection and $Y\ is\ \theta - T_2$ space, then $X\ is\ a\ simply - T_2$ space.

Proof: Let $Y\ is\ a\ \theta - T_2$space. For any distinct points $X\ and\ Y\ in\ X$. There exists disjoint $\theta - open\ set\ U\ and\ V\ in\ Y$ such that $f(X) \in U\ and\ f(Y) \in V$, Now f is faintly simply – continuous, then $f^{-1}(U)$, and $f^{-1}(V)$are simply - open in X containing $X\ and\ Y$respectively,

Thus $f^{-1}(U) \cap f^{-1}(V) = Q$ because$U \cap V = Q$.

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