A proposed Technique for Solving Interval-Valued Linear Fractional Bounded Variable Programming Problem

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Abstract

In this paper we premised an interval-valued linear fractional bounded variable programming problem (IVLFBP), by combining the interval-valued linear fractional programming problem (IVLFP) which is (LFP) with interval form coefficients in objective function, and linear fractional bounded variable problem (LFBV) which is (LFP) where the constraints are linear inequalities with bounded variables. Our method based on separating the main problem into two linear fractional bounded variable problems (LFBVP) and depends on the primal dual simplex algorithm. The algorithm that solves linear bounded variable programming will be extended to solve linear fractional with bounded variables, and then we used it to solve the interval-valued linear fractional bounded variable programming problems. We also compare our result with the solver function in the Microsoft Excel and matlab (R2011a)

Keywords: Interval-valued function, linear fractional programming, bounded variables (lower & upper bounds)

تقنية مقترحة لحل مشكلة البرمجة الكسرية بمعاملات مقيمة بفترات ومتغيراتِ محددة ِبفتراتَ

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الخلاصة

في هذا البحث قدّمنَا مشكلةَ البرمجة الكسرية بمعاملات مقيمة بفترات ومتغيراتِ محددةِبفتراتَ ايضا (IVLFBP)ا وذلك بدَمْج مشكلةِ البرمجة الكسرية بمعاملات مقيمة بفترات(IVLFP) والتي هي مشكلة برمجة كسرية خطيّةِ بمعاملاتِ الفترةِ في الدالة الموضوعيةِ ومشكلةِ البرمجة الكسرية محددات المتغير (LFBV) والتي هي برمجة كسرية خطيّةِ حيث المتغيرات تكون مقيدة بحدود فترات. طريقتنا مستندة على فَصْل المشكلةِ الرئيسيةِ إلى دالتين كسريتين بمتغيرات محددة بفترات وحل كل مشكلةِ على حدة معتمدين على الخوارزميةِ البسيطةِ الثنائيةِ الأساسيةِ. إنّ خوارزميةحلَ البرمجة الخطيّة بمتغيرات محددة بفترات يمكن انِ توسعُ لحَلّ المشكلةِالكسرية الخطيّة بمتغيرات محددةِ، وبعد ذلك إستعملنَاه لحَلّ مشاكل البرمجة الكسرية بمعاملات مقيمة بفترات ومتغيراتِ محددةِبفتراتَ .وقارنا نتيجتَنا ايضا بالنتائج التى حصلنا عليها باستخدام دالة (solver) في MS-Excel و ماتلاب (R2011a)

الكلمات المفتاحية: دالة بمعاملات مقيمة بفترات،برمجة كسرية خطية،متغيرات محددة (حدود سفلية وعلوية) .

Introduction

In different applications of non-linear programming, a ratio of two functions is to be maximized or minimized. In other applications the objective function may be consist of more than one such ratio. Ratio optimization in general is called fractional programming [1]. It has useful applications in financial and corporate planning, production planning, health care and hospital planning [2]. There are many economical, engineering and physical problems involve maximization (minimization) of the ratio of two functions, for example: cost/time, profit/cost or other quantities measuring the efficiency of the system [3]. A linear fractional programming problem (LFP) is a special case of fractional problems (FP) which is possible converting into linear (LFP) especially by Charnes and Cooper method [4]. Linear fractional programming (LFP) deals with that class of mathematical programming problem in which the relations among the variables are linear, the constraint relation must be in linear form and the objective function to be optimized must be a ratio of two linear functions, this field of LFP was developed by Hungarian mathematician B. Mtros in 1960 [5]. There are several methods to solve (LFP) such as the above-mentioned method of Charnes and Cooper in 1962[2], "updated objective function method" derived for solving (LFP) by Bitrain and Novaes 1973[6]. The above-mentioned methods and so many other methods that we can't mention them here dial with variables of type greater or equal to zero (≥ 0) [7]. While with the modeling of practical problem in real life, it found that one or more variables $x_j \geq 0$ and constrained by (upper or lower) bound or may be by both of them, specially there are optimization problems have probability the variables x_j of the model be inexact [8]. In such case all of the mentioned methods may fail. Beside this we discuss the problems with interval in coefficient and bounded variable which face as in real life that's why we try finding another procedure takes a little of computation effort. So, in this paper we will propose a new method for solving interval-valued linear fractional bounded variable Programming problems.

Preliminaries

Let I be the set of all bounded and closed intervals in the real numbers ($\mathbb{R}$), assume that $A, B \in I,\ then\ A, B\ will\ be\ write\ as\ A = \{a: a \in A\} = [a^L, a^U]$ And $B = \{b: b \in B\} = [b^L, b^U]$

Consider the following operations on ***I***

i) $A + B = [a^L, a^U] + [b^L, b^U] = [a^L + b^L, a^U + b^U] \in I$,
where both $a^L, a^U \in$ A and $b^L, b^U \in$ B

ii) –A=$[-a^U, -a^L] \in I$, where $a, -a \in A$

iii) from i) and ii) we have
$A - B$=$[a^L, a^U]$+$[-b^U, -b^L]$ = $[a^L - b^U, a^U - b^L] \in I$

iv) Let k$\in \mathbb{R}$. Then we can regard k as an interval [k, k] and then

$$kA=\{ka : a \in A\} = \begin{cases} [ka^L, ka^U] \in I & where\ k > 0 \\ [0,0] \in I & where\ k = 0 \\ [ka^U, ka^L] \in I & where\ k < 0 \end{cases}$$

Definition1: Let $A = [a^L, a^U]$ and $B = [b^L, b^U]$ be two nonempty bounded real intervals where $a, b \in \mathbb{R}$

i) We define the multiplication as

A*B=$[minimum(H),\ maximum(H)]$, $where\ H = \{a^L b^L, a^L b^U, a^U b^L, a^U b^U\}$

ii) $If\ 0 \notin [b^L, b^U]$ then we can define the division as follows:

A/B=$\frac{[a^L,a^U]}{[b^L,b^U]} = [minimum(S), maximum(S)]$, $where\ S = \left\{\frac{a^L}{b^U}, \frac{a^L}{b^L}, \frac{a^U}{b^U}, \frac{a^U}{b^L}\right\}$ such that

$\frac{1}{[b^L,b^U]} = \left[\frac{1}{b^U}, \frac{1}{b^L}\right]$ $where\ \ 0 \le b^L \le b^U$ and $\frac{[a^L,a^U]}{[b^L,b^U]} = [a^L, a^U]\frac{1}{[b^L,b^U]}$

$=[a^L, a^U]\left[\frac{1}{b^U}, \frac{1}{b^L}\right]$

However, we have to remember that "the quotient of two intervals is a set which may not be an interval". For *instance:* $\frac{\{1\}}{\{x:x\le 1\}} = \{\boldsymbol{x} : \boldsymbol{x} < 0\} \cup \{\boldsymbol{x} : \boldsymbol{1} \le \boldsymbol{x}\}$

There are different ways to express interval division [9].

Definition 2: consider the partial ordering $\preccurlyeq$ over I. $Let\ C = [c^L, c^U]$ $and\ D = [d^L, d^U]\ be$ *two closed intervals in* $\mathbb{R}$ *then we say that* C $\preccurlyeq$ D iff $c^L \le d^L$ and $c^U \le d^U$, now we write C $\prec$ D iff

$\begin{cases} c^L < d^L \\ c^U \le d^U \end{cases}$ OR $\begin{cases} c^L \le d^L \\ c^U < d \end{cases}$ OR $\begin{cases} c^L < d^L \\ c^U < d^U \end{cases}$

By the above definition we can interpret the concept of optimization of interval-valued function

Definition 3: An interval valued function is defined as $\boldsymbol{f}: \mathbb{R}^{\boldsymbol{n}} \rightarrow \boldsymbol{I}$

where $\mathbb{R}^{\boldsymbol{n}}$ *is* Euclidean space and $\boldsymbol{I}$ is the set of closed bounded intervals

(because $\boldsymbol{f}(\mathbf{x}) = \boldsymbol{f}(\boldsymbol{x_i})$, $(i = 1,2, \dots n)$ is closed interval in $\mathbb{R}$ for each $\mathbf{x} \in \mathbb{R}^{\boldsymbol{n}}$).

accorrding to the interval properities , we denote that $\boldsymbol{f}(\boldsymbol{x}) = [\boldsymbol{f}^{\boldsymbol{L}}(\boldsymbol{x}), \boldsymbol{f}^{\boldsymbol{U}}(\boldsymbol{x})]$

Where $\boldsymbol{f}^{\boldsymbol{L}}(\boldsymbol{x}) \le \boldsymbol{f}^{\boldsymbol{U}}(\boldsymbol{x})$ are real valued fractional functions and $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$

I. Standard form of linear programming (LP)

Linear programming may be defined as Maximizing (Minimizing) problem and it can be writing in a compact form as: $Max.(Min.)\ Z = CX$

$$S.t \qquad AX = b$$

$$X \ge 0,\ \ b \ge 0$$

where $A = \begin{pmatrix} a_{11}, a_{12} & \dots\dots a_{1n} \\ \vdots & \vdots \\ a_{m1}, a_{m2} & \dots\dots a_{mn} \end{pmatrix}$ *is* $m \times n$ *matrix*,

$b \in R^m$, $x = (x_1, x_2, \dots x_n)^T$ *and* C *is a* $(c_1, c_2, \dots, c_n)$

II. Standard form of linear fractional programming problem (LFP)

The formulation of linear fractional programming mathematically can be happening *as follows:*

$$Min.(Max.)Z = \frac{cx + \alpha}{dx + \beta}$$

$S.t \quad AX = b \quad , x \geq 0 \qquad (1)$

such that $A = (a_{ij})_{m \times n}$, *where* $b = (b_i)_{m \times 1} \in R^m$ and $X = (x_i)_{(n \times 1)}$ *are column vectors*

$c = (c_i)_{1 \times n}$, $d = (d_i)_{1 \times n}$ *are row vectors*

$\alpha, \beta \in \mathbb{R}$ and the denominator $dx + \beta \neq 0$,

$S = \{x \in R^n : Ax \leq b, x \geq 0\}$ is assumed to be the feasible region.

III. Formulation of the problem

Suppose that the following standard form of (LFP)

$$\text{Min. } z = \frac{cx + \propto}{dx + \beta} \qquad (2)$$

S.t $Ax = b$, $x \geq 0$,

Now suppose that $c = (c_j)$, $d = (d_j)$ Where $c_j, d_j \in I$ and $(j = 1,2, \dots, n)$.

we note that c_j^U & d_j^U the upper bounds of the c_j and d_j respectively it means that $c^U = (c_j^U)$ and also $d^U = (d_j^U)$ and similarly we can find c_j^L & d_j^L Where c_j and d_j are scalar real numbers and then $\alpha = [\alpha^L, \alpha^U]$, $\beta = [\beta^L, \beta^U]$

i.e. Minimize $z = \frac{[c_1^L, c_1^U]x_1 + [c_2^L, c_2^U]x_2 + [c_3^L, c_3^U]x_3 + \dots + [c_n^L, c_n^U]x_n + [\alpha^L, \alpha^U]}{[d_1^L, d_1^U]x_1 + [d_2^L, d_2^U]x_2 + [d_3^L, d_3^U]x_3 + \dots + [d_n^L, d_n^U]x_n + [\beta^L, \beta^U]}$

Subject to $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2\dots} & & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$, $\quad x_j \geq 0$,

Where $[d_1^L, d_1^U]x_1 + [d_2^L, d_2^U]x_2 + [d_3^L, d_3^U]x_3 + \dots + [\beta^L, \beta^U] \neq 0$

for all $x^T = (x_1, \ldots x_{n)} \in$

X where X be the "compact feasible region" of the problem above.

But we have to note that in this type of problem it may happen that one or more variables x_j satisfy the condition $0 \le l_j \le x_j \le u_j$ *where* $l_j, u_j \in \mathbb{R}$, so we can rewrite (2) as follows:

Minimize $z = \frac{p(y)}{q(y)} = \frac{cy+\gamma}{dy+\delta}$, (3)

Subject to $Ay = h$, $0 \le y \le u - l$,

Where l and u $\in \mathbb{R}$, x$\in \mathbb{R}^n$ *and* $x = y + l$, $\gamma = cl + \alpha$, $\delta = dl + \beta$ *and* $h = b - Al$

P(y), q(y) is interval valued linear functions as P(y) = $[p^L(y), p^U(y)]$ = $[c^L y + \gamma^L, c^U y + \gamma^U]$

And q(y) = $[q^L(y), q^U(y)]$ = $[d^L y + \delta^L, d^U y + \delta^U]$ and then the form (3) will be

IVBLFP (1) Minimize $z = \frac{p(y)}{q(y)} = \frac{[c^L y+\gamma^L, c^U y+\gamma^U]}{[d^L y+\delta^L, d^U x+\delta^U]}$,

Subject to $Ay = h$, (4)

$0 \le y \le u - l$

Now introducing of an "interval–valued linear fractional programming problem" can making by considering another type of possible "linear fractional programming problems" as follows:

IVBLFP (2) minimize f(y) = $[f^l(y), f^U(y)]$,

Subject to $Ay = h$, (5)

$0 \le y \le u - l$

Where f^L and f^U are linear fractional functions (as in 3).

Theorem1: Any *IVBLFP* in the form *IVBLFP (1)* under some assumption can be converting to the form *IVBLFP (2)*

Proof: The objective function in *IVBLFP (1)* is equal to $\frac{(p(y)}{q(y)}$ and *q(y)*$\neq 0$.

To convert the form (4) to the form (5), we assume that $0 \notin q(y)$.

For each feasible point y we should have

$0 < q^L(y) \le q^U(y)$ Or $q^L(y) \le q^U(y) < 0$

Using definition (1.ii), because the denominator doesn't contain zero we can formulate the objective function in (4) as:

$f(y) = [c^L y + \gamma^L, c^U y + \gamma^U][\frac{1}{d^U y+\delta^U}, \frac{1}{d^L y+\delta^L}]$ (6)

Now we can consider two cases and two possibilities states for each case as follows:

Case1: when 0≤ $q^L(y) \leq q^U(y)$, we have two possibilities:

i) When 0≤ $p^L(y) \leq p^U(y)$ using definition 1.i), we have

$f(y) = [\frac{c^L y+\gamma^L}{d^U y+\delta^U}, \frac{c^U y+\gamma^U}{[d^L y+\delta^L}]$ (7)

ii) When $p^L(y) \leq 0 \leq p^U(y), by\ definition$ 1. i) $we\ have$ $f(y) = [\frac{c^L y+\alpha^L}{d^L y+\beta^L}, \frac{c^U y+\alpha^U}{d^L y+\beta^L}]$ (8)

Case (2): When $q^L(y) \leq q^U(y) \leq 0$, we have two possibilities:

When 0≤ $p^L(y) \leq p^U(y)$ using definition1.i), we have

$f(y) = [\frac{c^U y+\alpha^U}{d^U y+\beta^U}, \frac{c^L y+\alpha^L}{d^L y+\beta^L}]$ (9)

ii) When $p^L(y) \leq 0 \leq p^U(y)$ using definition1.i), we have

$f(y) = [\frac{c^U y+\alpha^U}{d^U y+\beta^U}, \frac{c^L y+\alpha^L}{d^U y+\beta^U}]$ (10)

($Note$: since $p^L(y) \leq p^U(y) \leq 0$ is equivalent to $-p^L(y) \geq -p^U(y) \geq 0$)

$then$ the $subcase$ $p^L(y) \leq p^U(y) \leq 0$ is not difficult to derive from above cases.

Now according to definition (1.i), and considering above cases, the objective function in (4) can be formulate as in (5)

With respect to the corresponding cases which is mentioned above the require proof will be complete

Now following [10]. We interpret the meaning of minimization in (5);

Definition 4:

y^* *is called to be nondominated solution of problem* (5), if y^* *is a feasible solution of* the same *problem* and *there exist no feasible solution* y *such that* $f(y) \prec f(y^*)$ [10].

according to this case $f(y^*)$ *is said to be the nondominated objective value of* f.

$Now\ take\ the\ following\ optimization\ problem\ (corresponding\ to\ problem\ (5))$:

$$Min.\ g(y) = f^L(y) + f^U(y) \qquad (11)$$

S.t Ay=h

$$0 \le y \le u - l$$

We will use the following theorem to solve problem (5) from (1)

Theorem 2: If we have y^* as an optimal solution of problem (11), then y^* must be a nondominated solution of problem (5)

Proof: we note that the problem (11) and (3) has the identical feasible sets. Now assume that y^* is not a nondominated solution, Then there exist a feasible solution y such that $f(y) \prec f(y^*)$ from definition1.ii), it produce that $\begin{cases} f_L(y) < f_L(y^*) \\ f_U(y) \le f_U(y^*) \end{cases}$ Or $\begin{cases} f_L(y) \le f_L(y^*) \\ f_U(y) < f_U(y^*) \end{cases}$ or

$f_L(y) < f_L(y^*)$
$f_U(y) < f_U(y^*)$

It is also showing that $f(y) < f(y^*)$, which is contradict with the fact that $y^*$is an optimal solution of (11). Hence the proof is complete.

<u>The proposal technique algorithm</u>

Step1: we consider the original problem formula as in (2)

Minimize $z = \frac{[c_1{}^L, c_1{}^U]x_1 + [c_2{}^L, c_2{}^U]x_2 + [c_3{}^L, c_3{}^U]x_3 + \cdots + [c_n{}^L, c_n{}^U]x_n + [\alpha^L, \alpha^U]}{[d_1{}^L, d_1{}^U]x_1 + [d_2{}^L, d_2{}^U]x_2 + [d_3{}^L, d_3{}^U]x_3 + \cdots + [d_n{}^L, d_n{}^U]x_n + [\beta^L, \beta^U]}$

Subject to $\begin{pmatrix} a_{11} & a_{12} \ldots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2\ldots} & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$, $x_j \ge 0$,

Where $[d_1{}^L, d_1{}^U]x_1 + [d_2{}^L, d_2{}^U]x_2 + [d_3{}^L, d_3{}^U]x_3 + \cdots + [\beta^L, \beta^U] \ne 0$

Where the variables x_j satisfy the condition $0 \le l_j \le x_j \le u_j\ where\ l_j, u_j \in \mathbb{R}$,

Step2: Now we introduce other variables say y_j such that , $0 \le y_j \le u_j - l_j$,

Where l_j and $u_j \in \mathbb{R}$, x$\in \mathbb{R}^n\ and\ x_j = y_j + l_j$

And then the problem will be of the form (3)

Step3: According to (4) we convert the problem to the form of ratio of two interval functions

$f(y) = \frac{[c^L y + \gamma^L, c^U y + \gamma^U]}{[d^L y + \delta^L, d^U x + \delta^U]}$

Step4: Now according to the (Theorem1) the problem will of the form of function interval $f(y) = [f^L(y), f^u(y)]$ consist of linear fractional program as lower and upper bounds

Step5: use the following procedure to solve each of $f^u(y)$ and $f^L(y)$ separately

i) Multiplying the right-hand side by (-1) if it is negative

ii) Insert "slack and surplus" variable to convert the (LFBV) to the standard form

iii) Compute $Z_1 = C_B Y_B \text{value} + \alpha$ and $Z_2 = D_B Y_B \text{value} + \beta$ and then $P(y) = \frac{Z_1}{Z_2}$

iv) Compute $\Delta_j = z_2 * (c_j - z_{j1}) - z_1 * (d_j - z_{j2})$ where $z_{j1} = C_B a_J$ and $z_{j2} = D_B a_J$

v) For Minimization "optimum basic feasible solution is attained if all $\Delta_j \geq 0$ for all non-basic variables at their lower bound" and for (Maximization) "optimum basic feasible solution is attained if all $\Delta_j \leq 0$ for all non-basic variables at their lower bound". if not go to next step

vi) For Maximization select most positive Δ_j (For Minimization select most negative Δ_j)

vii) Let y_i be the non-basic variable at zero level which selected to enter the solution.

viii) Select the intersection cell of (a_j) the column of most positive Δ_j, with the row of minimum ratio (bj/aj), then the corresponding basic variable S_j will departure.

ix) We will continue until all $\Delta_j \leq 0$ for maximization (all $\Delta_j \geq 0$ for minimization) then we reach the optimal solution.

Numerical examples

Example 1: $Minimize\ z = \frac{[-3,-1]x_1+[2,4]x_2+[-2,-.5]}{[.5,1.5]x_1+[.5,1.5]x_2+[3,5]}$,

Subject to: $-x_1 + x_2 \leq 2$,

$2x_1 + 3x_2 \leq 14$,

$x_1 - x_2 \leq 5$,

$4 \leq x_1 \leq 6 \quad 0 \leq x_2 \leq 1$,

By equation (4) the example will be

$Minimize\ z = \frac{[-3x_1+2x_2-2, -x_1+4x_2-.5]}{[.5x_1+.5x_2+3\ , 1.5x_1+1.5x_2+5\ ]}$ with the above constraint Then by the sub-case that refer to $p^L(y) \leq p^U(y) \leq 0$ is equivalent to $-p^L(y) \geq -p^U(y) \geq 0)$

produce that $Minimize\ z = [\frac{-3x_1+2x_2-2}{.5x_1+.5x_2+3}, \frac{-x_1+4x_2-.5}{1.5x_1+1.5x_2+5}]$ with the above constraints also, Now if $x_1 = y_1 + 4$, $0 \le y_1 \le 2$, and $x_2 = y_2$, $0 \le y_2 \le 1$,

Then $Minimize = \left[\frac{-3y_1+2y_2-14}{.5y_1+.5y_2+5}, \frac{-y_1+4y_2-4.5}{1.5y_1+1.5y_2+11}\right] = [f^L(y), f^U(y)]$

Subject to: $-y_1 + y_2 \le 6$, $2y_1 + 3y_2 \le 6$, $y_1 - y_2 \le 1$,

Where $0 \le y_1 \le 2$, $0 \le y_2 \le 1$,

Such that $f^L(y) = \frac{-3y_1+2y_2-14}{.5y_1+.5y_2+5}$ and $f^U(y) = \frac{-y_1+4y_2-4.5}{1.5y_1+1.5y_2+11}$,

Now $f^L(y) = \frac{-3y_1+2y_2-14}{.5y_1+.5y_2+5}$ with the above constraint

Table 1: Initial Table

cB ↓	dB ↓	CJ →		-3	2	0	0	0	Ratio (bj/aj)	
		DJ →		0.5	0.5	0	0	0		
		Basic Variable	Solution(b)	y1	y2	S1	S2	S3		
0	0	s1	6	-1	1	1	0	0	-6	
0	0	s2	6	2	3	0	1	0	3	
0	0	s3	1	1*	-1	0	0	1	1	←
z1	-14	zj1		0	0	0	0	0		
z2	5	zj2		0	0	0	0	0		
z	-2.8	cj-zj1		-3	2	0	0	0		
		dj-zj2		0.5	0.5	0	0	0		
		Δj →		-8	17	0	0	0		
				↑						

Table 2: nondominated solution of $f^L(y)$

cB ↓	dB ↓	→ CJ		-3	2	0	0	0	Ratio bj/aj
		→ DJ		0.5	0.5	0	0	0	
		Basic Variable	Solution (b)	y1	y2	S1	S2	s3	
0	0	s1	7	0	0	1	0	1	undefined
0	0	s2	4	0	5	0	1	-2	1.8
-3	0.5	y1	1	1	-1	0	0	1	-1
z1	-17	zj1		-3	3	0	0	-3	
z2	5.5	zj2		0.5	-0.5	0	0	0.5	
z	-3.091	cj-zj1		0	-1	0	0	3	
		dj-zj2		0	1	0	0	-0.5	
		Δj →		0	115	0	0	8	

i.e. $y_1 = 1 \Longrightarrow x_1 = 5$ and $y_2 = 0 \Longrightarrow x_2 = 0$

Also $f^U(y) = \frac{-y_1+4y_2-4.5}{1.5y_1+1.5y_2+11}$ with same constraint

Table 3: Initial Table

cB	dB	CJ		-1	4	0	0	0	Ratio bj/aj
		DJ		1.5	1.5	0	0	0	
		Basic Variable	Solution(b)	y1	Y2	S1	S2	s3	
0	0	s1	6	-2	2	1	0	-1	-3
0	0	s2	6	1	4	0	1	-1	6
0	0	s3	1	1*	-1	0	0	1	1
z1	-4.5	zj1		0	0	0	0	0	
z2	11	zj2		0	0	0	0	0	
z	-0.409	cj-zj1		-1	4	0	0	0	
		dj-zj2		1.5	1.5	0	0	0	
		Δj →		-4.25	50.75	0	0	0	

Table 4: Nondominate solution of $f^U(y)$

cB	dB	CJ		-1	4	0	0	0	
		DJ		1.5	1.5	0	0	0	
		Basic Variable	Solution(b)	y1	y2	S1	S2	s3	Ratio
0	0	s1	8	0	0	1	0	1	
0	0	s2	-8	0	5	0	1	-2	
-1	1.5	y1	1	1	-1	0	0	1	
z1	-5.5	zj1		-1	1	0	0	-1	
z2	12.5	zj2		1.5	-1.5	0	0	1.5	
z	-0.44	cj-zj1		0	3	0	0	1	
		dj-zj2		0	3	0	0	-1.5	
		Δj →		0	46.5	0	0	4.25	

again $y_1 = 1 \Longrightarrow x_1 = 5$ and $y_2 = 0 \Longrightarrow x_2 = 0$ with objective value $f = (-3.09, -0.44)$

Example 2: $Maximize\ z = \frac{[1,2]x_1+[2,3.5]x_2+[3,4]x_3+[1,3]x_4}{[.5,1]x_2+[.25,2]x_3+[2.5,4]x_4+[1,3]}$

Subject to $3x_1 + 2x_2 + x_3 + 4x_4 \le 8$

$5x_1 + 3x_2 + 2x_3 + 5x_4 \le 15$

$2x_1 - x_2 + 4x_3 + 6x_4 \le 2$

$0 \le x_1 \le 3$, $1 \le x_2 \le 2$, $2 \le x_3 \le 4$ $0 \le x_4 \le 1$

By equation (4) the example will be of the form

Max $z = \frac{[x_1+2x_2+3x_3+x_4, 2x_1+3.5x_2+4x_3+3x_4]}{[.5x_2+.25x_3+2.5x_4+1, x_2+2x_3+4x_4+3]}$ with the above constraints.

Then by (7) it will be Max $z = [\frac{x_1+2x_2+3x_3+x_4}{x_2+2x_3+4x_4+3}, \frac{2x_1+3.5x_2+4x_3+3x_4}{.5x_2+.25x_3+2.5x_4+1}]$ with the same constraints.

Now suppose that $x_1 = y_1\ where\ \ 0 \le y_1 \le 3$, $x_2 = y_2 + 1\ where\ \ 0 \le y_2 \le 1,$

$x_3 = y_3 + 2\ where\ \ 0 \le y_3 \le 2$

$and\ x_4 = y_4\ where\ \ 0 \le y_4 \le 1$, then the problem is changed to the form

Max $z = \left[\frac{y_1+2y_2+3y_3+y_4+8}{y_2+2y_3+4y_4+8}, \frac{2y_1+3.5y_2+4y_3+3y_4+11.5}{.5y_2+.25y_3+4y_4+2}\right] = [f^L(y), f^U(y)]$

Subject to $3y_1 + 2y_2 + y_3 + 4y_4 \le 4$

$5y_1 + 3y_2 + 2y_3 + 5y_4 \le 8$

$2y_1 - y_2 + 4y_3 + 6y_4 \le -5$

$0 \le y_1 \le 3$, $0 \le y_2 \le 1$, $0 \le y_3 \le 2$, $0 \le y_4 \le 1$

Now we solve $f^L(y) = \frac{y_1+2y_2+3y_3+y_4+8}{y_2+2y_3+4y_4+8}$ with the above constraints

Table 1: Initial table

CB ↓	DB ↓	CJ →		1	2	3	1	0	0	0		
		DJ →		0	1	2	4	0	0	0		
		Basic Variable	solution	y1	y2	y3	y4	S1	S2	s3	Ratio	
0	0	s1	4	3	2	1	4	1	0	0	4	
0	0	s2	8	5	3	2	5	0	1	0	4	
0	0	s3	-5	2	-1	4*	6	0	0	1	-1.25	←
z1	8	zj1		0	0	0	0	0	0	0		
z2	8	zj2		0	0	0	0	0	0	0		
z	1	cj-zj1		1	2	3	1	0	0	0		
		dj-zj2		0	1	2	4	0	0	0		
		Δ →		8	8	8	-24	0	0	0		
						↑						

Table 2:

CB ↓	DB ↓	CJ →		1	2	3	1	0	0	0		
		DJ →		0	1	2	4	0	0	0		
		Basic Variable	solution	y1	y2	y3	y4	S1	S2	s3	Ratio	
0	0	s1	5.25	2.5	2.25*	0	2.5	1	0	-0.25	2.33333	←
0	0	s2	10.5	4	3.5	0	2	0	1	-0.5	3	
3	2	x3	-1.25	0.5	-0.25	1	1.5	0	0	0.25	5	
z1	4.25	zj1		1.5	-0.75	3	4.5	0	0	0.75		
z2	5.5	zj2		1	-0.5	2	3	0	0	0.5		
z	0.77	cj-zj1		-0.5	2.75	0	-3.5	0	0	-0.75		
		dj-zj2		-1	1.5	0	1	0	0	-0.5		
		Δ →		1.5	8.75	0	-24	0	0	-2		
					↑							

Table 3: Nondominate solution of $f^L(y)$

CB	DB	CJ →		1	2	3	1	0	0	0	
↓	↓	DJ →		0	1	2	4	0	0	0	
		B Variable	solution	y1	y2	y3	y4	S1	S2	s3	Ratio
2	1	y2	2.333	1.111	1	0	1.111	0.444	0	-0.111	2.33333
0	0	s2	7	0.5	0	-3.5	-1.5	-3.5	-2.5	-4	Undef.
3	2	y3	-0.667	0.778	0	1	1.778	0.111	0	0.222	Undef.
z1	10.67	zj1		2.333	0	3	5.333	0.333	0	0.667	
z2	9	zj2		1.556	0	2	1.778	0.111	0	0.222	
z	1.185	cj-zj1		-1.333	2	0	-4.333	-0.333	0	-0.667	
		dj-zj2		-1.56	1	0	.444	-0.222	0	-0.444	
		Δ →		-.72	6.75	0	-26	-.89	0	-1.78	

It means $x_1 = 0$, $x_2 = 2.333 + 1 = 3.333$, $x_3 = -.667 + 2 = 1.333$ *and* $x_4 = 0$

Also, we solve $f^U(y) = \frac{2y_1+3.5y_2+4y_3+3y_4+11.5}{-y_1+.5y_2+y_3+4y_4+3.5}$ with the same constraints

Table 1: Initial table

CB	DB	CJ →		2	3.5	4	3	0	0	0	
↓	↓	DJ →		0	0.5	0.25	2.5	0	0	0	
		Bivariable	solution	y1	y2	y3	y4	S1	S2	s3	ratio
0	0	s1	4	3	2	1	4	1	0	0	4
0	0	s2	8	5	3	2	5	0	1	0	4
0	0	s3	-5	2	-1	4*	6	0	0	1	-1 ←
z1	11.5	zj1		0	0	0	0	0	0	0	
z2	2	zj2		0	0	0	0	0	0	0	
z	5.75	cj-zj1		2	3.5	4	3	0	0	0	
		dj-zj2		0	0.5	0.25	2.5	0	0	0	
		Δ →		4	1.3	5.13	-23	0	0	0	
						↑					

Table 2:

CB	DB	CJ →		2	3.5	4	3	0	0	0	
↓	↓	DJ →		0	0.5	0.25	2.5	0	0	0	
		B. Variable	solution	y1	y2	y3	y4	S1	S2	s3	ratio
0	0	y1	5.25	2.5	2.3*	0	2.5	1	0	-0	2.3 ←
0	0	s2	10.5	4	3.5	0	2	0	1	-1	3
4	0.25	s3	-1.25	0.5	-0	1	1.5	0	0	0.3	5
z1	6.5	zj1		2	-1	4	6	0	0	1	
z2	1.69	zj2		0.1	-0	0.25	0.4	0	0	0.1	
z	3.85	cj-zj1		0	4.5	0	-3	0	0	-1	
		dj-zj2		-0	0.6	0	2.1	0	0	-0	
		Δ →		0.8	3.9	0	-19	0	0	-1	
					↑						

Table 3: Nondominate solution of $f^U(y)$

CB	DB	CJ →		2	3.5	4	3	0	0	0	
↓	↓	DJ →		0	0.5	0.25	2.5	0	0	0	
		B. Variable	solution	y1	y2	y3	y4	S1	S2	s3	ratio
3.5	0.5	x2	2.333	1.1	1	0	1.1	0.4	0	-0	2.3
0	0	s2	2.333	0.1	0	0	-2	-2	1	-0	###
4	0.25	x3	-0.667	0.8	0	1	1.8	0.1	0	0.2	###
z1	5.5	zj1		3.1	0	4	7.1	0.4	0	0.9	
z2	3	zj2		0.2	0	0.25	0.4	0	0	0.1	
z	1.83	cj-zj1		-1	3.5	0	-4	-0	0	-1	
		dj-zj2		-0	0.5	0	2.1	-0	0	-0	
		Δ →		-1	2.7	0	-20	-1	0	-1	

Again $x_1 = 0$, $x_2 = 3.333$, $x_3 = 1.333$ and $x_4 = 0$ With Max z= [1.185,1.83]

Note: We solved several examples by this procedure such as:

Example 3:$Minimize\ z = \frac{[3,5]x_1+[1,4]x_2+[7,11]}{[\frac{1}{2},2]x_1+[1,2]x_2+[4,6]}$

Subject to $x_1 + 3x_2 \leq 30$, $-x_1 + 2x_2 \leq 5$ where $20 \leq x_1 \leq 35$, $x_2 \geq 0$

"Optimal solution is $x_1 = 30$, $x_2 = 0$" and the objective function value $f = [\frac{97}{66}, \frac{161}{19}]$

Conclusion

In this paper, at first we combined the interval valued fractional linear function with fractional linear function with bounded variable and then we introduce two possible type of equations (4) and (5) and then we converted the bounded variable to another non negative variable and we convert equation (4) to the form of (5),at last we solve it as two separable equations , also we solved each problem with Matlab (R2011a) and solver function in MS-Excel and the results obtained are same.

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