Solving a Class of High Order Non-linear Partial Differential Systems Using Adomian Decomposition Method

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Abstract

In this paper, Adomian Decomposition Method with Adomain Polynomials are proposed to solve a class of high-order non-linear partial differential systems. The method is applied to non-linear fifth-order Mikhailov-Novikov-Wang and sixth-order Coupled Ramani Systems. This method provides an accurate and efficient technique in comparison with other classical methods, the solutions procedure are very simple and in few iteration leads to high accurate solutions. The numerical results indicated that the obtained approximate solutions were in suitable agreement with the exact solutions.

Keywords: Adomian Decomposition Method, Mikhailov-Novikov-Wang system, Coupled Ramani System.

حل صنف من أنظمة المعادلات التفاضلية الجزئية اللا خطية ذات الرتب العليا باستخدام طريقة Adomian للتحلل

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الخلاصة

في هذا البحث، طريقة تحلل Adomian اقترحت لحل صنف من انظمة المعادلات التفاضلية الجزئية ذات الرتب العليا اللاخطي. الطريقة طبقت لنظام Mikhailov-Novikov-Wang اللاخطي من الرتبة الخامسة ونظام Ramani اللاخطي من الرتبة السادسة. هذه الطريقة توفر تقنية دقيقة وكفوء بالمقارنة مع الطرق التقليدية الاخرى مع اجراءات الحلول بسيطة جدا مع تكرارات قليلة تؤدي الى حلول دقيقة وعالية، حيث اظهرت النتائج العددية ان الحلول التقريبية كانت قريبة جدا من الحل المضبوط.

الكلمات المفتاحية: طريقة تحلل Adomian، نظام Mikhailov-Novikov-Wang غير الخطي، نظام Coupled Ramani غير الخطي.

Introduction

George Adomian was exposed and developed method so-called a decomposition method for "solving differential equation, integro-differential, differential-delay and partial differential equations" [14,15]. The solution is found as an infinite sequence which converges rapidly to accurate solutions. This strategy has been demonstrated fruitful in dealing with both linear as well as non-linear problems.The technique solution of a non-linear operator equation is consist on a decomposition equations in a sequences of functions. Every sequence for each expression is acquired from a polynomial produced from an expansion of an analytic function into a power sequences.The ADM (Adomian Decomposition Method) strategy is exceptionally straightforward in an a theoretical detailing yet the trouble emerges in calculating the polynomials and in demonstrating the convergence of the series of functions [16]. Lesnic [11] examined the convergence of the ADM when utilized time-dependent heat, wave equations for both forward and backward time evolution. In [5], El-Sayed and Gaber solving PDEs of fractal order in a finite domain using the Adomian technique. Ghoreishi, et. al. [21] studied ADM to solve non-linear wave-like equations with variable coefficients and they demonstrate that ADM can be capable to solve this type of equations with no requirement for dissertation, perturbation, transformation and linearization.

In [1] Cheniguel and Ayadi presented the ADM for solving non-homogenous heat equation and they offer that this technique is an efficient of the other classical strategies. Developmen a strategy ADM is "submited for the solution of the generalized fifth-order Korteweg-de-Vries (GFKDV) equation" [18]. Al-Rozbayani [3] applied ADM to calculate numerical solution of Allen-Cahn equation. In [4] Al-Eybani assumes strategy of ADM and the differential transform method to estimate approximate and exact solution of the heat equation with a power non-linearity. Likewise, the comparison technique of ADM and differential quadratic strategy for solving some non-linear PDEs is introduced by Firoozjai and Yazdani [24]. In [22] the advance technique of ADM is applied to solve linear and non-linear of BVPs with Neumann condition.

In this study let us assume the class of fifth – order systems as [10]:

$$\begin{aligned} &u_t + \rho u_{xxxxx} + \gamma u u_{xxx} + \beta u_x u_{xx} + \alpha u^2 u_x - w_x = 0 \\ &w_t + 6 w u_{xxx} + 2 w_x u_{xx} - 96 w u u_x - 16 w_x u^2 = 0 \end{aligned} \qquad (1)$$

Such that α, β, γ and ρ are real arbitrary nonzero parameters and $u = u(x,t), w = w(x,t)$ are differentiable functions. List of forms of (1) can be reformulated by variation the values of parameters.

Specialty status where $\alpha = -80, \beta = 50, \gamma = 20,$ and $\rho = -1$,then the formula of equations (1) can be reduces to the Mikhailov-Novikov-Wang system (MVW) [8,9]

$$\begin{aligned} &u_t = u_{xxxxx} - 20 u u_{xxx} - 50 u_x u_{xx} + 80 u^2 u_x + w_x \\ &w_t = -6 w u_{xxx} - 2 w_x u_{xx} + 96 w u u_x + 16 w_x u^2 \end{aligned} \qquad (2)$$

Which has been derived and employed the symmetric approach by the authors in [7].

The second model of high-order non-linear PDEs is named Coupled Ramani Equations which can be formulated as following [2, 13]:

$$u_{xxxxxx}+15u_{xx}u_{xxx}+15u_{x}u_{xxxx}+45u_x^2u_{xx}-5\left(u_{xxxt}+3u_{xx}u_t+3u_xu_{xt}\right)-5u_{tt}+18v_x=0$$
$$v_t-v_{xxx}-3v_xu_x-3vu_{xx}=0 \qquad (3)$$

"Multi-soliton solutions of the coupled Ramani equations were derived and represented considering Pfaffians in a compact form in [19] and the three-soliton solution of this coupled system is examined by Hu et. al. [23]".

The exact solutions of the "coupled Ramani equations" were determined using the Tanh approach in [12,13,20]. In [6] the Hirota's bilinear method was used to forecast multiple soliton solutions and multiple singular soliton solutions for Ramani equation. The coupled Ramani equations are examined by means of an analytic technique, namely the Homotopy analysis method (HAM) [2].

This paper is organized as follows. In section 2, the basic concept of the ADM was described. Section 3, was devoted to solve a class of fifth-order non-linear systems by ADM. Section 4, the sixth-order Coupled Ramani system was solved by ADM and concluding observations are given in section 5.

Adomian Decomposition Technique (ADM)

Adomian decomposition method it is a capable technique using for solving non-linear functional equations. This technique is depends on reformulation a non-linear functional equation as a sequence of functions, such that every particular of this sequence is represent by a polynomial created by a power sequences expansion of an analytic function and it is exceptionally easy in a resume formulation. The trouble emerges appear in determining the polynomials and in demonstrating the convergence of the sequences of functions. Assuming that general non-linear coupled of PDEs composition in an operator's form [17]:

$$L_1(u)+R_1(u,v)+N_1(u,v)=G_1(x,t)$$
$$L_2(v)+R_2(u,v)+N_2(u,v)=G_2(x,t) \qquad (4)$$

Undergo to initial conditions:

$$
\begin{aligned}
&u(x,0) = f_0(x), && v(x,0) = g_0(x) \\
&\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = f_1(x), && \left.\frac{\partial v(x,t)}{\partial t}\right|_{t=0} = g_1(x) \\
&\vdots \\
&\left.\frac{\partial u^{s-1}(x,t)}{\partial t^{s-1}}\right|_{t=0} = f_{s-1}(x), && \left.\frac{\partial v^{s-1}(x,t)}{\partial t^{s-1}}\right|_{t=0} = g_{s-1}(x)
\end{aligned} \tag{5}
$$

Where $L = \frac{\partial^s}{\partial t^s}$, $s = 1,2,3,\cdots$ represent highest partial derivative w.r.t t, $R_1(u,v)$, $R_2(u,v)$ are a linear operators, $N_1(u,v)$, $N_2(u,v)$ are the non-linear terms. $G_1(x,t)\, and\, G_2(x,t)$ are specific functions.

The inverse operator L^{-1} is an integral operator, which is given by:

$$L^{-1}(.) = \int_0^t \int_0^t \cdots\cdots \int_0^t (.)\,dt \cdots\cdots dt\,dt \tag{6}$$

Applying L^{-1} on equation (4) and using the restrictions lead to:

$$
\begin{aligned}
u(x,t) &= f_0(x) + f_1(x)t + \cdots + f_{s-1}(x)t^{s-1} + L^{-1}(G_1) - L^{-1}(R_1(u,v) + N_1(u,v)) \\
v(x,t) &= g_0(x) + g_1(x)t + \cdots + g_{s-1}(x)t^{s-1} + L^{-1}(G_2) - L^{-1}(R_2(u,v) + N_2(u,v))
\end{aligned} \tag{7}
$$

The ADM proposes that the linear functions u(x,t) and v(x,t) can be decomposed by an infinite sequences of components:

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \qquad v(x,t) = \sum_{n=0}^{\infty} v_n(x,t), \tag{8}$$

Moreover, the linear and non-linear operators $R_1, R_2,\, N_1\, and\, N_2$ by the infinite sequences can be written as:

$$R_1(u,v) = \sum_{n=0}^{\infty} A_n, \quad N_1(u,v) = \sum_{n=0}^{\infty} B_n, \quad R_2(u,v) = \sum_{n=0}^{\infty} C_n, \quad N_2(u,v) = \sum_{n=0}^{\infty} D_n, \tag{9}$$

Such that $u_n(x,t)$ *and* $v_n(x,t), n \geq 0$ represent the components of u(x,t) and v(x,t) that will be suitable setting and A_n, B_n, C_n *and* D_n are called Adomian's polynomials. For instance, if $R_1(u)$ is from one variable and $N_1(u,v)$ is non-linear from tow variables u and v, so A_n, B_n can be setting as:

$$
\begin{aligned}
A_n(u_0,\cdots,u_n) &= \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[R_1\left(\sum_{i-0}^{n}\lambda^i u_i\right)\right]_{\lambda=0} \\
B_n(u_0,\cdots,u_n;v_0,\cdots,v_n) &= \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N_1\left(\sum_{i=0}^{n}\lambda^i u_i,\sum_{i=0}^{n}\lambda^i v_i\right)\right]_{\lambda=0}
\end{aligned}
\qquad (10)
$$

According to the method of decomposition [17], the non-linear system (4) can be realize in a formula of the following recursive relations:

$$
\begin{aligned}
u_0(x,t) &= f_0(x) + f_1(x)t + \cdots + f_{s-1}(x)t^{s-1} + L^{-1}(G_1) \\
v_0(x,t) &= g_0(x) + g_1(x)t + \cdots + g_{s-1}(x)t^{s-1} + L^{-1}(G_2)
\end{aligned}
\qquad (11)
$$

And

$$
\begin{aligned}
u_{n+1}(x,t) &= -L^{-1}\left[\sum_{n=0}^{\infty}A_n + \sum_{n=0}^{\infty}B_n\right], \\
v_{n+1}(x,t) &= -L^{-1}\left[\sum_{n=0}^{\infty}C_n + \sum_{n=0}^{\infty}D_n\right] \qquad n \geq 0,
\end{aligned}
\qquad (12)
$$

It is clearly seeing that the components of zeroth u_0 *and* v_0 are known, then the residual components u_n *and* $v_n, n \geq 1$, can be wholly calculated by such way that every component is determined by using the preceding conditions. The outcome, the components $u_0, u_1, u_2, \cdots$ *and* $v_0, v_1, v_2 \cdots$, are specified and the sequences solutions are perfectly resolved. With regard to comparison numerically, let use setting the solution u(x,t) and v(x,t) as follows:

$$
\lim_{n\to\infty}\Phi_n(x,t) = u(x,t), \qquad \lim_{n\to\infty}\Psi_n(x,t) = v(x,t),
$$

Where

$$\Phi_n(x,t)=\sum_{k=0}^{n}u_k(x,t), \qquad \Psi_n(x,t)=\sum_{k=0}^{n}v_k(x,t), \quad n\geq 0 \tag{13}$$

The iteration relation is assumed in (11) and (12). Furthermore, the decomposition sequences solutions mostly converged quickly for veritable physical problems.

Gb m;

Application ADM to the Solution MNW System

In this section, we implement the steps to obtain a numerical solution of non-linear fifth-order system (1) using Adomian decomposition method with the initial conditions [8,10]:

- (14)

The differential operator is defined as:

$$L_t(.)=\frac{\partial}{\partial t}(.)$$

And the inverse operator L_t^{-1} exists, so it can putting as:

$$L_t^{-1}(.)=\int_0^t(.)\,dt$$

Now, from equation (4), the fifth-order system (1) becomes:

$$\begin{aligned}L_t(u)+R_1(u,w)+N_1(u)&=0\\ L_t(w)+N_2(u,w)&=0\end{aligned} \tag{15}$$

Such that

$$\begin{aligned}R_1(u,w)&=\rho u_{xxxxx}-w_x\\ N_1(u)&=\gamma u u_{xxx}+\beta u_x u_{xx}+\alpha u^2 u_x\\ N_2(u,w)&=6wu_{xxx}+2w_x u_{xx}-96wuu_x-16w_x u^2=0\end{aligned}$$

Now, applying the inverse operator on both sides of (15) and using the initial conditions (14), yields:

$$u(x,t) = f(x) - L_t^{-1}\left(R_1(u,w) + N_1(u)\right)$$
$$w(x,t) = g(x) - L_t^{-1}\left(N_2(u,w)\right)$$

using (12), we obtain:

$$u(x,t) = f(x) - L_t^{-1}\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} B_n\right],$$
$$w(x,t) = g(x) - L_t^{-1}\left[\sum_{n=0}^{\infty} D_n\right] \qquad (16)$$

The Adomian polynomials A_n, B_n and D_n are produced with respect to (10), we can permit the first few Adomian polynomials of A_n, B_n and D_n respectively:

$$A_0 = \rho \frac{\partial^5 u_0}{\partial x^5} - \frac{\partial w_0}{\partial x}$$
$$A_1 = \rho \frac{\partial^5 u_1}{\partial x^5} - \frac{\partial w_1}{\partial x}$$
$$A_2 = \rho \frac{\partial^5 u_2}{\partial x^5} - \frac{\partial w_2}{\partial x}$$
$$\vdots$$

And

$$B_0 = \gamma u_0 \frac{\partial^3 u_0}{\partial x^3} + \beta \frac{\partial u_0}{\partial x}\frac{\partial^2 u_0}{\partial x^2} + \alpha u_0^2 \frac{\partial u_0}{\partial x}$$
$$B_1 = \gamma\left(u_0 \frac{\partial^3 u_1}{\partial x^3} + u_1 \frac{\partial^3 u_0}{\partial x^3}\right) + \beta\left(\frac{\partial u_0}{\partial x}\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial u_1}{\partial x}\frac{\partial^2 u_0}{\partial x^2}\right) + \alpha\left(u_0^2 \frac{\partial u_1}{\partial x} + 2u_0 u_1 \frac{\partial u_0}{\partial x}\right)$$
$$\vdots$$

And

$$D_0 = 6\, w_0 \frac{\partial^3 u_0}{\partial x^3} + 2\, \frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_0}{\partial x} - 96\, w_0 u_0 \frac{\partial u_0}{\partial x} - 16\, u_0^2 \frac{\partial w_0}{\partial x}$$

$$D_1 = 6\left(w_0 \frac{\partial^3 u_1}{\partial x^3} + w_1 \frac{\partial^3 u_0}{\partial x^3} \right) + 2\left(\frac{\partial^2 u_0}{\partial x^2}\frac{\partial w_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2}\frac{\partial w_0}{\partial x} \right)\ldots$$
$$-96\left(w_0\, u_0 \frac{\partial u_1}{\partial x} + w_0\, u_1 \frac{\partial u_0}{\partial x} + w_1\, u_0 \frac{\partial u_0}{\partial x} \right) - 16\left(2u_0\, u_1 \frac{\partial w_0}{\partial x} + u_0^2 \frac{\partial w_1}{\partial x} \right)$$
$$\vdots$$

And so on.

According to (11) and (14), the zeroth components u_0 *and* w_0 written as follows:

$$u_0(x,0) = \frac{\sqrt{\lambda}}{2} + \frac{3}{4}\sqrt{\lambda}\tan^2\left(\lambda^{\frac{1}{4}}\, x\right)$$

$$w_0(x,0) = \frac{\lambda^{\frac{3}{2}}}{6}$$

Now, the recursive relation can be putting as:

$$u_1(x,t) = \frac{-1}{\cos^7\left(\lambda^{\frac{1}{4}}\, x\right)} (2*10^{-9}\, \lambda^{\frac{7}{4}} \sin\left(\lambda^{\frac{1}{4}}\, x\right) t\, (2.7*10^{11}\, \rho + 3.8962*10^{9} + 4.2188*10^{8}\, \alpha + 3.375*10^{9}\, \beta$$
$$+1.2*10^{10}\, \rho \cos^4\left(\lambda^{\frac{1}{4}}\, x\right) + 4.6875*10^{7}\, \alpha \cos^4\left(\lambda^{\frac{1}{4}}\, x\right) - 1.8*10^{11}\, \rho \cos^2\left(\lambda^{\frac{1}{4}}\, x\right)$$
$$-2.25*10^{9}\, \beta \cos^2\left(\lambda^{\frac{1}{4}}\, x\right) - 2.8125*10^{8}\, \alpha \cos^2\left(\lambda^{\frac{1}{4}}\, x\right) - 2.5975*10^{9} \cos^2\left(\lambda^{\frac{1}{4}}\, x\right)$$
$$+4.3291*10^{8} \cos^4\left(\lambda^{\frac{1}{4}}\, x\right) + 2 \cos^6\left(\lambda^{\frac{1}{4}}\, x\right)))$$

$$w_1(x,t) = \frac{1}{\cos^5\left(\lambda^{\frac{1}{4}}\, x\right)} (6\, \lambda^{\frac{11}{4}} \sin\left(\lambda^{\frac{1}{4}}\, x\right) (\cos^2\left(\lambda^{\frac{1}{4}}\, x\right) - 3) - \frac{1}{\cos^5\left(\lambda^{\frac{1}{4}}\, x\right)} (6\, \lambda^{\frac{11}{4}} \sin\left(\lambda^{\frac{1}{4}}\, x\right) (\cos^2\left(\lambda^{\frac{1}{4}}\, x\right) - 3)$$
$$= 0$$

Then

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots\cdots$$

$$u(x,t)=-\frac{\sqrt{\lambda}}{4\cos^2\left(\lambda^{\frac{1}{4}}x\right)}\quad(\cos^2\left(\lambda^{\frac{1}{4}}x\right)-3)-\frac{1}{\cos^7\left(\lambda^{\frac{1}{4}}x\right)}(2*10^{-9}\,\lambda^{\frac{7}{4}}\sin\left(\lambda^{\frac{1}{4}}x\right)\,t\,(2.7*10^{11}\,\rho+3.8962*10^{9}$$

$$+4.2188*10^{8}\,\alpha+3.375*10^{9}\,\beta+1.2*10^{10}\,\rho\,\cos^4\left(\lambda^{\frac{1}{4}}x\right)+4.6875*10^{7}\,\alpha\,\cos^4\left(\lambda^{\frac{1}{4}}x\right)$$

$$-1.8*10^{11}\,\rho\,\cos^2\left(\lambda^{\frac{1}{4}}x\right)-2.25*10^{9}\,\beta\,\cos^2\left(\lambda^{\frac{1}{4}}x\right)-2.8125*10^{8}\,\alpha\,\cos^2\left(\lambda^{\frac{1}{4}}x\right)$$

$$-2.5975*10^{9}\,\cos^2\left(\lambda^{\frac{1}{4}}x\right)+4.3291*10^{8}\,\cos^4\left(\lambda^{\frac{1}{4}}x\right)+2\,\cos^6\left(\lambda^{\frac{1}{4}}x\right)))+\cdots\cdots$$

and

$$w(x,t)=w_0(x,t)+w_1(x,t)+w_2(x,t)+\cdots\cdots$$

$$w(x,t)=\frac{\lambda^{\frac{3}{2}}}{6}$$

Which gives exact solution of w (x,t) [8]. The numerical output are listed in table (1).

Table 1: Absolute error of u(x,t) with $\alpha=-80,\beta=50,\gamma=20,\lambda=0.001,\rho=-1$ and t=0.1

x	ADM	Exact solution	Absolute error
-1	0.01656396598	0.01657733127	1.3366 e-005
-0.8	0.01628763892	0.01629781642	1.0178 e-005
-0.6	0.01607601359	0.01608335938	7.3458 e-006
-0.4	0.01592696904	0.01593173384	4.7648 e-006
-0.2	0.01583904003	0.01584138357	2.3436 e-006
0.2	0.01584378721	0.01584144367	2.3435 e-006
0.4	0.01593661944	0.01593185465	4.7648 e-006
0.6	0.01609088791	0.01608354214	7.3458 e-006
0.8	0.01630824050	0.01629806301	1.0178 e-005
1	0.01659100954	0.01657764427	1.3365 e-005

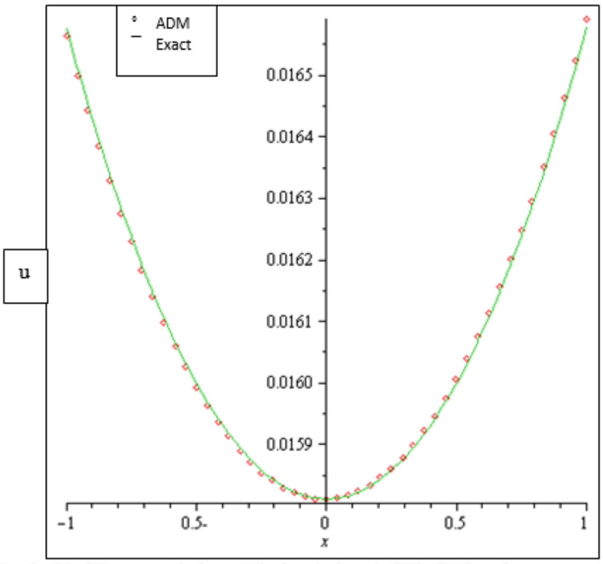

Figure 1: Comparison between Exact results for u and ADM approximate results for **MNW System when** $\alpha=-80, \beta=50, \gamma=20, \lambda=0.001$, $\rho=-1$ **and** t=0.1

Application ADM to Coupled Ramani System

In this section, we employ ADM to obtain a numerical solution of non-linear sixth-order system (3) with respect to initial conditions [2, 13]:

$$u(x,0)=a_0-2\alpha\ \tan(\alpha x)$$
$$u_t(x,0)=2\beta\alpha^2\sec^2(\alpha x)$$
$$v(x,0)=-\left(\frac{4}{9}\right)\beta\alpha^4+\left(\frac{16}{27}\right)\alpha^6-\left(\frac{5}{9}\right)\beta^2\alpha^2-\left(\frac{5}{54}\right)\beta^3+\left(-\left(\frac{20}{9}\right)\beta\alpha^4+\left(\frac{16}{9}\right)\alpha^6-\left(\frac{5}{9}\right)\beta^2\alpha^2\right)\tan^2(\alpha x) \tag{17}$$

Now, from equation (4), the sixth-order system (3) becomes:

$$L_{tt}(u)+R_1(u,v)+N_1(u)=0$$
$$L_t(v)+R_2(v)+N_2(u,v)=0 \tag{18}$$

s.t

$$R_1(u,v)=\frac{1}{5}\frac{\partial^6 u}{\partial x^6}-\frac{\partial^4 u}{\partial x^3\,\partial t}+\frac{18}{5}\frac{\partial v}{\partial x}$$

$$N_1(u)=3\frac{\partial^2 u}{\partial x^2}\frac{\partial^3 u}{\partial x^3}+3\frac{\partial u}{\partial x}\frac{\partial^4 u}{\partial x^4}+9\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial^2 u}{\partial x^2}-3\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial t}-3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\,\partial t}$$

$$R_2(v)=\frac{\partial^3 v}{\partial x^3}$$

$$N_2(u,v)=3\frac{\partial v}{\partial x}\frac{\partial u}{\partial x}+3v\frac{\partial^2 u}{\partial x^2}$$

By taking the inverse operator on both sides of (18) and applying the initial conditions, we obtain:

$$u(x,t)=f(x)+h(x)t-L_{tt}^{-2}\left(R_1(u,v)+N_1(u)\right)$$
$$v(x,t)=g(x)-L_t^{-1}\left(R_2(v)+N_2(u,v)\right)$$

By using (12), we obtain:

$$u(x,t)=f(x)+h(x)t-L_{tt}^{-2}\left[\sum_{n=0}^{\infty}A_n+\sum_{n=0}^{\infty}B_n\right],$$

$$v(x,t)=g(x)-L_t^{-1}\left[\sum_{n=0}^{\infty}C_n+\sum_{n=0}^{\infty}D_n\right] \qquad (19)$$

From equation (10), the first few Adomian polynomials of A_n, B_n *and* D_n respectively are:

$$A_0=\frac{1}{5}\frac{\partial^6 u_0}{\partial x^6}-\frac{\partial^4 u_0}{\partial x^3\,\partial t}+\frac{18}{5}\frac{\partial v_0}{\partial x}$$

$$A_1=\frac{1}{5}\frac{\partial^6 u_1}{\partial x^6}-\frac{\partial^4 u_1}{\partial x^3\,\partial t}+\frac{18}{5}\frac{\partial v_1}{\partial x}$$

$$\vdots$$

,

$$B_0=3\frac{\partial^2 u_0}{\partial x^2}\frac{\partial^3 u_0}{\partial x^3}+3\frac{\partial u_0}{\partial x}\frac{\partial^4 u_0}{\partial x^4}+9\left(\frac{\partial u_0}{\partial x}\right)^2\frac{\partial^2 u_0}{\partial x^2}-3\frac{\partial^2 u_0}{\partial x^2}\frac{\partial u_0}{\partial t}-3\frac{\partial u_0}{\partial x}\frac{\partial^2 u_0}{\partial x\,\partial t}$$

$$B_1=3\frac{\partial^2 u_0}{\partial x^2}\frac{\partial^3 u_1}{\partial x^3}+3\frac{\partial^2 u_1}{\partial x^2}\frac{\partial^3 u_0}{\partial x^3}+3\frac{\partial u_0}{\partial x}\frac{\partial^4 u_1}{\partial x^4}+3\frac{\partial u_1}{\partial x}\frac{\partial^4 u_0}{\partial x^4}+9\left(\frac{\partial u_0}{\partial x}\right)^2\frac{\partial^2 u_1}{\partial x^2}+18\frac{\partial u_0}{\partial x}\frac{\partial u_1}{\partial x}\frac{\partial^2 u_0}{\partial x^2}$$
$$-3\frac{\partial^2 u_0}{\partial x^2}\frac{\partial u_1}{\partial t}-3\frac{\partial^2 u_1}{\partial x^2}\frac{\partial u_0}{\partial t}-3\frac{\partial u_0}{\partial x}\frac{\partial^2 u_1}{\partial x\,\partial t}-3\frac{\partial u_1}{\partial x}\frac{\partial^2 u_0}{\partial x\,\partial t}$$
$$\vdots$$

,

$$C_0=\frac{\partial^3 v_0}{\partial x^3}$$
$$C_1=\frac{\partial^3 v_1}{\partial x^3}$$
$$\vdots$$

,

$$D_0=3\frac{\partial v_0}{\partial x}\frac{\partial u_0}{\partial x}+3v_0\frac{\partial^2 u_0}{\partial x^2}$$
$$D_1=3\frac{\partial v_0}{\partial x}\frac{\partial u_1}{\partial x}+3\frac{\partial v_1}{\partial x}\frac{\partial u_0}{\partial x}+3v_0\frac{\partial^2 u_1}{\partial x^2}+3v_1\frac{\partial^2 u_0}{\partial x^2}$$
$$\vdots$$

And so on.

As the same process in the above section when we use (11) and (17), the zeroth components u_0 *and* v_0 can be written as follows:

$$u_0(x,0)=a_0-2\alpha\ \tan(\alpha x)+2\ t\,\beta\,\alpha^2\sec^2(\alpha x)$$
$$v_0(x,0)=-\left(\frac{4}{9}\right)\beta\,\alpha^4+\left(\frac{16}{27}\right)\alpha^6-\left(\frac{5}{9}\right)\beta^2\alpha^2-\left(\frac{5}{54}\right)\beta^3+\left(-\left(\frac{20}{9}\right)\beta\,\alpha^4+\left(\frac{16}{9}\right)\alpha^6-\left(\frac{5}{9}\right)\beta^2\alpha^2\right)\tan^2(\alpha x)$$

Where the recursive relation of u(x,t) and v(x,t) can also be written as:

$$u_1(x,t)=-\frac{2}{15\cos^{10}(\alpha x)}(\alpha^3\beta t^2(15\beta\sin(\alpha x)\cos^7(\alpha x)+150\alpha^3\beta t\ \cos^4(\alpha x)-120\alpha^3\beta t\ \cos^6(\alpha x)$$
$$-48\alpha^5 t\ \cos^6(\alpha x)+32\alpha^5 t\ \cos^8(\alpha x)-648\alpha^7\beta^2 t^3+1080\alpha^7\beta^2 t^3\ \cos^2(\alpha x)$$
$$-432\alpha^7\beta^2 t^3\ \cos^4(\alpha x)-1440\alpha^6\beta t^2\sin(\alpha x)\cos(\alpha x)+1800\alpha^6\beta t^2\sin(\alpha x)\cos^3(\alpha x)$$
$$-480\alpha^6\beta t^2\sin(\alpha x)\cos^5(\alpha x)))$$

$$v_1(x,t)=\frac{1}{9\cos^6(\alpha x)}(\alpha^3 \beta t(768\beta t \cos^2(\alpha x)-600\alpha^5 \beta t-150\alpha^3 \beta^2 t+128\alpha^7 t \cos^4(\alpha x)$$
$$+480\alpha^7 t-192\alpha^5 \beta t \cos^4(\alpha x)+10\alpha \beta^3 t \cos^4(\alpha x)+40\alpha^2 \beta \sin(\alpha x)\cos^3(\alpha x)$$
$$+120\alpha^3 \beta^2 t \cos^2(\alpha x)-15\alpha \beta^3 t \cos^2(\alpha x)-32\alpha^4 \sin(\alpha x)\cos^3(\alpha x)-576\alpha^7 t \cos^2(\alpha x)$$
$$+10 \beta^2 \sin(\alpha x)\cos^3(\alpha x))$$

Then

$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots\cdots$$

$$u(x,t)=-\frac{1}{15\cos^{10}(\alpha x)}(960\alpha^9 \beta^2 t^4 \sin(\alpha x)\cos^5(\alpha x)+15\cos^{10}(\alpha x)$$
$$+240\alpha^6 \beta^2 t^3 \cos^6(\alpha x)+30\alpha^2 \beta t \cos^8(\alpha x)-64\alpha^8 \beta t^3 \cos^8(\alpha x)$$
$$-30\alpha \sin(\alpha x)\cos^9(\alpha x)+2880\alpha^9 \beta^2 t^4 \sin(\alpha x)\cos(\alpha x)+1296\alpha^{10} \beta^3 t^5$$
$$-2160\alpha^{10} \beta^3 t^5 \cos^2(\alpha x)+864\alpha^{10} \beta^3 t^5 \cos^4(\alpha x)-3600\alpha^9 \beta^2 t^4 \sin(\alpha x)\cos^3(\alpha x)$$
$$-300\alpha^6 \beta^2 t^3 \cos^4(\alpha x)+96\alpha^8 \beta t^3 \cos^6(\alpha x)-30\alpha^3 \beta^2 t^2 \sin(\alpha x)\cos^7(\alpha x))+\cdots\cdots$$

$$v(x,t)=v_0(x,t)+v_1(x,t)+v_2(x,t)+\cdots\cdots$$

$$v(x,t)=-\frac{1}{54\cos^6(\alpha x)}(120\alpha^4 \beta \cos^4(\alpha x)+3600\alpha^8 \beta^2 t^2-2880\alpha^{10} \beta t^2+900\alpha^6 \beta^3 t^2$$
$$+5\beta^3 \cos^6(\alpha x)+64\alpha^6 \cos^6(\alpha x)-96\alpha^4 \beta \cos^6(\alpha x)+1152\alpha^8 \beta^2 t^2 \cos^4(\alpha x)$$
$$+768\alpha^{10} \beta t^2 \cos^4(\alpha x)-60\alpha^4 \beta^4 t^2 \cos^4(\alpha x)+30\alpha^2 \beta^2 \cos^4(\alpha x)$$
$$+3456\alpha^{10} \beta t^2 \cos^2(\alpha x)-96\alpha^6 \cos^4(\alpha x)+192\alpha^7 \beta t \sin(\alpha x)\cos^3(\alpha x)$$
$$-240\alpha^5 \beta^2 t \sin(\alpha x)\cos^3(\alpha x)-60\alpha^3 \beta^3 t \sin(\alpha x)\cos^3(\alpha x)$$
$$-4608\alpha^8 \beta^2 t^2 \cos^2(\alpha x)-720\alpha^6 \beta^3 t^2 \cos^2(\alpha x)+90\alpha^4 \beta^4 t^2 \cos^2(\alpha x))+\cdots\cdots$$

The numerical results are listed in tables (2) and (3).

Table 2: Absolute error of u (x, t) with $\alpha = 0.01, \beta = 0.01, a_0 = 1$ and t=20

x	ADM	Exact solution	$\lvert u_{exact} - u_{hom}\rvert$ [2]	$\lvert u_{exact} - u_{ADM}\rvert$
-50	1.010978045	1.010978045	3.5228 e-009	1.4241 e-010
-40	1.008503055	1.008503055	9.9889 e-009	1.0716 e-010
-30	1.006230580	1.006230580	4.1012 e-008	8.2320 e-011
-20	1.004095862	1.004095861	3.4089 e-007	6.7190 e-011
-10	1.002047105	1.002047104	1.4429 e-005	5.7584 e-011
10	0.9980337015	0.9980337012	2.0652 e-005	5.8560 e-011
20	0.9959874264	0.9959874261	3.5501 e-007	6.7625 e-011
30	0.9938570760	0.9938570755	4.1594 e-008	7.9838 e-011
40	0.9915912458	0.9915912460	1.0076 e-008	6.7625 e-011
50	0.9891258312	0.9891258314	3.5466 e-009	5.9416 e-011

Table 3: Absolute error of v (x, t) with $\alpha = 0.01, \beta = 0.01, a_0 = 1$ and t=20

x	ADM	Exact solution	$\lvert u_{exact} - u_{hom}\rvert$ [2]	$\lvert u_{exact} - u_{ADM}\rvert$
-50	-9.99322755 e-008	-9.99322721 e-008	4.3539 e-013	3.4220 e-015
-40	-9.92360419 e-008	-9.92360399 e-008	1.6285 e-012	1.8960 e-015
-30	-9.87525637 e-008	-9.87525626 e-008	9.0038 e-012	1.1090 e-015
-20	-9.84342473 e-008	-9.84342466 e-008	1.0129 e-010	7.0270 e-016
-10	-9.82525133 e-008	-9.82525128 e-008	6.4273 e-009	5.1120 e-016
10	-9.82478304 e-008	-9.82478298 e-008	6.3932 e-009	5.3648 e-016
20	-9.84244956 e-008	-9.84244948 e-008	1.0023 e-010	7.5790 e-016
30	-9.87369021 e-008	-9.87369009 e-008	8.8634 e-012	1.2048 e-015
40	-9.92130132 e-008	-9.92130112 e-008	1.5952 e-012	2.0520 e-015
50	-9.98994982 e-008	-9.98994945 e-008	4.2446 e-013	3.6760 e-015

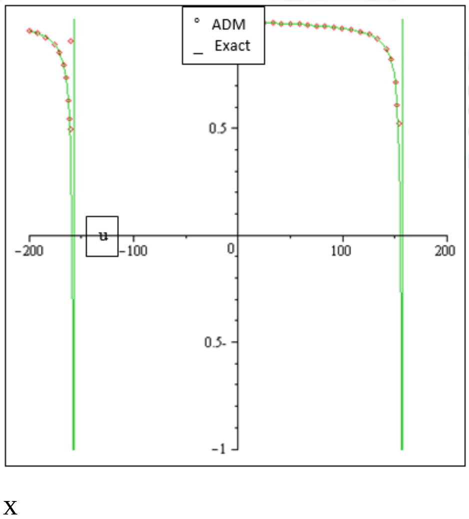

Figure 2: Comparison between Exact results for u and ADM approximate results for Coupled Ramani System when $\alpha = 0.01, \beta = 0.01, a_0 = 1$ and t=20

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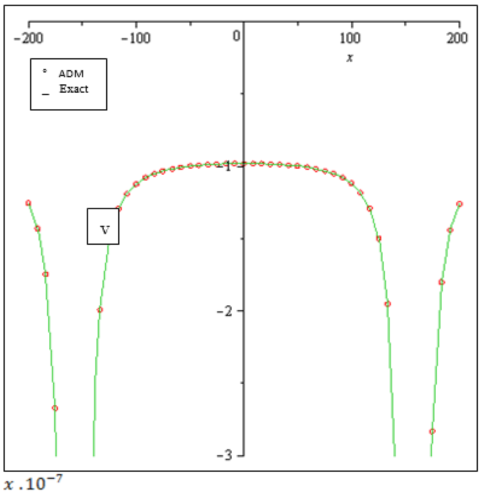

Figure 3: Comparison between Exact results for v and ADM approximate results for Coupled Ramani System when $\alpha = 0.01, \beta = 0.01, a_0 = 1$ and t=20

Conclusions

In this paper, we solve a class of high-order non-linear partial differential systems by using “Adomian decomposition method”. ADM is quantitative rather than qualitative, analytic, requiring neither linearization nor perturbation, and continuous with no resort to discretization and consequent computer-intensive calculations. The main advantage of this technique is that it be enabled utilized directly for all type of "differential and integral equations". On the other hand, worthy advantage is that the ADM is qualified for decreasing the size of computational work while still maintaining high accuracy of the numerical solution. Also, the numerical results show that the ADM has perfect approximation effect and high accuracy for solving high-order non-linear PDEs.We observed from the numerical results in tables (2) and (3) that the ADM provides more precise and robust numerical solution than the Homotopy analysis method proposed by [2]. Finally, the computation in this paper has been done using the MAPLE 13 software package.

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