Bornological Group

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Abstract

In this paper we introduce the concept of Bornological Group, G-space and study some of their properties.

Keyword: Bornological space, bounded set, and bounded linear map.

الزمرالبرنولوجية

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الملخص

ضمن هذا البحث عرفنا مفهوم الزمر البرنولوجية وقدمنا تعريفا لفضاءG ودراسة بعض المفاهيم الاولية عنهما.

الكلمات المفتاحية : الفضاء البرنولوجي ، المجموعات المحددة ،

Introduction

For the first time in (1977), H. Hogbe–NIend [4] introduced the Concept of Bornology on a set and study BornologicalConstruction.In (1981)M.D.Patwardhan[5]extended this idea to space of entire functions. However in (2005) [1] studied Bornological **Constructions** of the space of entire functions and included a new Bornological Construction by using many concepts such as, the bases for the Bornology. In (2009) [2] Al-Salihi, Anwar, NoorAl-Deen Introduced the concept of Bornology on a semi-bounded set and introduce semi-bounded Linear map, semi convergence in Bornological space. In (2011) [3] introduced the concept of semi-unbounded map in Bornological space and product space. In this paper we introduce the concept of Bornological Group. A Bornological Group is a mathematical object with both an algebraic structure and a Bornological structure. Thus, one may perform algebraic operations,because of the group structure,and one may talk about bounded linear map, because of the Bornology. Bornological Group with bounded group actions, are used to study bounded symmetries which have many applications for example in physics. In the language of category theory, Bornological groups can be defined concisely as group objects in the category of Bornological space, in the same way that ordinary groups are group objects in the category theory of sets. Not that the axioms are given in terms of the maps (binary product and cutlery identity), hence are categorical definition.Through this paper we assume (X,β) to be Bornological space.

2-Preliminaries

In this section we recall several basic notions from theory of bounded set in Bornological space.

(2-1) Definition:

A Bornology on a set X is a family β of subsets of X satisfying the following axioms: β is a convering of X. (ii) **β** is hereditary under inclusion)**i**(

(iii)βis stable under finite union. (X,β) consisting of a set X and a Bornology β on X called a **Bornological Space** , and the elements of β are called bounded sets.[4].

(2-2)Example

Let $X=R^n$ with Euclidean norm i.e .Then the collection β= {A ⊆R^n :A is bounded subset of R^n in usual sense} is a bornology on R^n called the canonical Bornology (or usual Bornology) on R^n, In the fact a closed D_r ={x∈ R^n: a subset A of R^n bounded iff there exists a closed disk D with center 0 s.t. A⊆D.

(i)It is clear that every closed disk in R^n is bounded set i.e. D∈β, since R^n is covered by family of all closed disks i.e. β is covered R^n.

(ii) If A∈ β and K⊆A, then∃closed disk such that K⊆ A⊆ D. There for$K$ ∈ β.

(iii) if $A_1,A_2,\ldots,An\in$ β,then∃$D_{r1},\ldots,D_{r2}$ are closed disks s.t $A_1\subseteq D_{r1},\ldots,A_n\subseteq D_{rn}$ There forβis stable under finite union. (R^n,β) consisting of a set R^n and a Bornology β onR^n called a **Bornological Space**.

(2-3)Definition

Let X and Y be two Bornological spaces and U:X→Y. We say that U is a bounded map if the image under U of every bounded subset of X is Bounded in Y i.e U(A) ∈B(Y) ∀ A∈ B(X) [4].

3-Bornological Group

In this section we present the definition of Bornological Groups and some fundamental definition of G-space.

(3-1)Definition

A Bornological Group is a set G with two structures

1-G is a group

2-G is Bornological space

such that the two structures are compatible, i.e. they satisfy the following conditions

(i)-The multiplication map U:G*G →G such that $U(g_1,g_2)= g_1g_2$ for each $g_1g_2\in G$ is bounded.

(ii)The map V:G→ G such that V(g)=g for each g∈G is bounded.

In this definition the set G*G carries the product Bornology.

(3-2) Example

R^+ with respect to multiplication with the usual Bornology is a Bornological Group.

(i)R^+ is a group with respect to(., -1, e)

(ii)R^+ is a Bornological space The two structures are compactible i.e the multiplication map U: B*B→B and the map V:B→ B bouth bounded.

(3-3)Definition

A homorphism between two Bornological Group G and H is abounded group homorphism F:G→H.

(3-4)Definition

An isomorphsm in Bornological Groups is a bounded group homorphism having an inverse which is also a bounded group homorphism .

(3-5)Definition

A Bornological transformation group is a triple (G,X,π) where G is aBornological group, X is a Bornological space and π:G*X→ X is bounded linear map such that

(i)$\pi\left(g_1 {}_{,} \pi(g_2,x)\right) = \pi(g_1g_2\,,x)$for all $g_1g_2 \in G \; and \; x \in X.$

(ii)$\pi(e,x) = x$ for all $x \in X$ where e the identity element of G.

The function is called an action of G onX. The space X togather withπ is called a G-space.

(3-6)Example :

G=R^+ by (3-2) is a Bornological group X=R with the usual Bornology is a Bornological space.

Define π :R^+ *R →R by π(t,x)=x+$\ln t$ for each $x \in R, t \in R^+$

(i)$\pi(1, x) = x + \ln 1 = x$

(ii) $\pi(t_1, \pi(t_2, x)) = \pi(t_1, x + \ln t_2) = x + \ln(t_1 t_2) = \pi(t_1 t_2, x)$.

Where $t_1, t_2 \in R^+$Hence R is an R^+ -space.

(3-7)Definition

Let X be a G-space , A⊆X. Thenasubset A of X is invariant under G if GA⊆A where GA={ga| g∈ G, a∈ A}.

(3-8)Example :

(Z,+) with the discrete Bornolgy is a Bornological group, R^2 with the usual Borology. Define Q:Z*R^2→ R^2 by Q(n,(x,y)=(n+x,n+y) for each n∈ Z ,(x,y) ∈ R^2 thus :

1- Q is bounded

2-Q(0,(x,y)=(x,y).

3-Q(n_1, Q(n_2,(x,y)=Q(n_1,(n_2+x,n_2+y))=(n_1+n_2+x,n_1+n_2+y)=Q(n_1+n_2,(x,y)) for each n_1,n_2∈ Z. Hence R^2 is Z-space.

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