Investigation of the ground state features for some Sn-isotopes in the framework of Skyrme-Hartree-Fock method

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Abstract

The nuclear ground state properties of some Sn-isotopes have been investigated by the Skyrme-Hartree-Fock (SHF) method with the Skyrme parameters; SII, SIII, SV, SKXce, SLy4 and SKT. These nuclear properties include the charge, proton and matter densities and their root mean square (rms) radii, neutron skin thickness, nuclear binding energy per particle and charge form factor. The obtained results are compared with the available measured data and the relativistic mean field theory (RMFT) results.

Keywords: Skyrme–Hartree–Fock (SHF) method, Sn-isotopes, Skyrme parameters

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دراسة خصائص الحالة الارضية لبعض نظائر القصدير باستخدام طريقة سكيرم- هارتري- فوك

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الخلاصة

في هذه الدراسة تمت دراسة خصائص الحالة الارضية لبعض نظائر القصدير باستخدام طريقة سكيرم- هارتري- فوك مع برامترات مختلفة هي SII, SIII, SV, SKXce, SLy4, SKT. هذه الخصائص تشمل كثافة كل من الشحنة ، البروتون والكتلة مع انصاف الاقطار المرافقة لها، السمك النيوتروني، معدل طاقة الربط النووية و عوامل التشكل. لقد تمت مقارنة النتائج النظرية مع نظيراتها من القيم العملية المتاحة والنتائج النظرية المحسوبة باستخدام نظرية معدل المجال النسبي.

الكلمات المفتاحية: طريقة سكيرم- هارتري- فوك، نظائر القصدير، باراميترات سكيرم.

Introduction

The Skyrme-Hartree-Fock (SHF) method is widespread used to study the features of nuclei. This method is successfully used for a wide range of nuclear properties such as neutron and proton density, root mean square (rms) charge radii, binding energy, etc. The Hartree-Fock description of nuclear characteristics gives good results not only for stable spherical and deformed nuclei, but also for neutron-deficient and neutron-rich nuclei [1-3].One of the main quantities used to describe the nuclear properties is the nuclear charge density distribution. The charge densities directly related to the wave functions of protons, therefore they are can be given us much detailed information on the internal structure of nuclei. Charge density distributions for stable nuclei have been good studied with electron-nucleus scattering method which is known to be one of the powerful tools for investigating nuclear charge density distributions [4,5].Tel *et al.* [3] have been used the Hartree-Fock method to calculate the ground-state properties for the neutron- rich isotopes of Boron and compared the calculated results with the experimental data. Huseyin *et al.* [6] studied the neutron, proton and charge densities, neutron, proton, mass and charge rms radii and neutron skin thickness for the some neutron-rich isotopes by the Hartree-Fock method. Shen and Ren [7] have been study the ground-state properties of He, Li, and Be nuclei using Skyrme-Hartree-Fock model with force parameters MSKA. They have been successfully reproduced neutron halo in nuclei ^{6}He, ^{8}He, ^{11}Li, and ^{14}Be. In this research, we investigate the applicability of the SHF method with the Skyrme parameters; SII [8], SIII [2], SV [9], SKXce [10], SLy4 [11] and SKT [12] to study the static nuclear properties for ^{116,118,119,120,122,124}Sn stable nuclei and compared the obtained results with the results of relativistic mean field theory (RMFT) and the available experimental data.

Theory

The Skyrme force is an effective interaction with a two-body and three body parts [2]:

$$V_{CS} = \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)} \qquad (1)$$

The Skyrme forces with the three-body term replaced by a density-dependent two-body term are unified in a single form [12] and used for the central potential as an extended Skyrme force [13]:

$$V_{Skyrme} = \sum_{i<j} V_{ij} = \hat{V}^{m} + \hat{V}^{L.S} + \hat{V}^{t} \tag{2}$$

where

$$\hat{V}^{m} = t_0\left(1 + x_0\hat{P}_\sigma\right)\delta_{12} + \frac{t_3}{6}\left(1 + x_3\hat{P}_\sigma\right)\rho^{\alpha}(r_1)\delta_{12} + \frac{t_1}{2}\left(1 + x_1\hat{P}_\sigma\right)\left[\delta_{12}\hat{k}^2 + \hat{k}'^2\delta_{12}\right] + t_2\left(1 + x_2\hat{P}_\sigma\right)\hat{k'}.\delta_{12}\hat{k} \tag{3}$$

$$\hat{V}^{L.S} = it_4(\hat{\sigma}_1 + \hat{\sigma}_2).\hat{k} \times \delta_{12}\vec{k}' \tag{4}$$

$$\hat{V}^{t} = \frac{t_0}{2}\left\{\left(3\left(\hat{\sigma}_1.\hat{k}\right)\left(\hat{\sigma}_2.\hat{k}\right) - (\hat{\sigma}_1.\hat{\sigma}_1)\hat{k}'^2\right)\delta_{12} + \delta_{12}\left(3\left(\hat{\sigma}_1.\hat{k}\right)\left(\hat{\sigma}_2.\hat{k}\right) - (\hat{\sigma}_1.\hat{\sigma}_1)\hat{k}^2\right)\right\} + t_0\left(3\left(\hat{\sigma}_1.\hat{k'}\right)\delta_{12}\left(\hat{\sigma}_2.\hat{k}\right) - (\hat{\sigma}_1.\hat{\sigma}_1)\hat{k}'.\delta_{12}\hat{k}\right) \tag{5}$$

Where $\hat{V}^{m}$ the tensor force. It is interesting to note that the Skyrme interaction is particularly restrictive with respect to the spin–orbit terms $\hat{V}^{L.S}$, $\hat{P}_\sigma$ is the spin exchange operator and $\hat{\sigma}$ is the spin operator and t_0, t_1, t_2, t_3, t_4, x_0, x_1, x_2, x_3, and α are Skyrme force parameters,$\delta_{12} = \delta(r_1 - r_2)$ and the $\hat{k}$ and $\hat{k}'$ operators are the relative wave vectors of two nucleons acts to the right and to the left (*i.e.* the complex conjugate wave functions, with coordinate r'), respectively. They have the form:

$$\hat{k} = \left(\nabla_i - \nabla_j\right)/2i \tag{6}$$

$$\hat{k'} = -\left(\nabla_i - \nabla_j\right)/2i \tag{7}$$

the neutron and proton densities are given by the following relation [14]:

$$\rho_k(\vec{r}) = \sum_{\beta\in k} w_\beta \psi_\beta^{+}(\vec{r})\, \psi_\beta(\vec{r}), \tag{8}$$

where k denotes the proton or neutron, ψ_β is the single-particle wave function of the state β and w_β represents the occupation probability of the state β.

The charge form factor, $F_k(q)$, where q is the momentum transfer, is obtained from the nuclear charge density by the Fourier–Bessel transform [15]:

$$F_k(q) = 4\pi \int_0^\infty r^2 j_0(qr)\,\rho_k(r)\,dr \tag{9}$$

The root mean square (rms) radii of the neutron, proton and charge distributions can be obtained from these densities as follows [2]

$$r_k = \langle r_k^2 \rangle^{1/2} = \left[\frac{\int r^2 \rho_k(r)dr}{\int \rho_k(r)dr}\right]^{1/2} \qquad k = n, p, c \tag{10}$$

The neutron skin thickness *t*, is defined as:

$$t = r_n - r_p \tag{11}$$

Results and Discussions

The ground state properties of Sn isotopes, such as charge, proton and mass densities and corresponding root mean square radius have been calculated using Skyrme-Hartree-Fock (SHF) method with certain Skyrme force parameters namely; SII, SIII, SV, SKXce, SLy4 and SKT are listed in table-1. For completeness, elastic charge form factors and the binding energies per nucleon are evaluated within the same framework.

Table-1: Skyrme force parameters used in the present investigation.

Parameter	SII [8]	SIII [2]	SV [9]	SKXce [10]	SLy4 [11]	SKT [12]
t_0 (MeV.fm^3)	-1169.9	-1128.75	-1248.29	-1437.96	-2488.91	-1788.9
t_1 (MeV.fm^5)	586.6	395	970.56	244.335	486.820	301.5
t_2 (MeV.fm^5)	-27.1	-95	107.22	-133.741	-546.39	502.5
t_3 (MeV.fm$^{3\alpha}$)	9331.1	14000	0	12116.264	13777	12764
t_4 (MeV.fm^5)	105	120	150	145.706	123	130
x_0	0.34	0.45	-0.17	0.289	0.834	0.353
x_1	0	0	0	0.611	-0.344	-2.5
x_2	0	0	0	0.145	-1	-1.7
x_3	1	1	1	-0.056	1.354	0.475
α	1	1	1	0.5	0.167	0.333

The calculated charge rms radii for considered nuclei along with the experimental data [16] and those of relativistic mean field theory (RMFT) [17] results are tabulated in table-2 and shown in fig. 1(a). It can be shown that the calculations by the SHF method reveal a good agreement with the experimental data especially, theoretical calculations with SKXce parameter. The calculated results based on the different parameters shown in this table, decrease with increasing the neutron number from 4.603-4.678 fm for (^{116}Sn) to 4.648- 4.735 fm for (^{124}Sn). In addition, the obtained values of charge rms radii calculated with SKT parameter are in good accordance with the RMFT results.The proton, neutron and matter rms radii for Sn- isotopes obtained by different Skyrme parameters are given in tables 3-5 and demonstrated in figures 1(b) -1(d), respectively. The calculated results of the proton and neutron rms radii are compared with the relativistic mean field theory (RMFT) results [17]. One can notice from this comparison that the calculated proton rms radii with SKT parameter are more close to the RMFT results than other parameters, while the evaluated results of neutron rms radii with the all parameters are smaller than the results of RMFT. Furthermore, it can be seen from figs. 1(b) -1(d) that these radii increase with increasing the nucleon number. In addition, the obtained neutron skin thickness *t* values with SLy4 parameter are given in table 4. Note that the neutron skin thickness *t* values have been increase from 0.098 fm for (^{116}Sn) to 0.175 fm for (^{124}Sn) by increasing the neutron number.

Table-2: Comparison of the calculated charge rms radii (in fm) Sn isotopes with the experimental data

Nuclei	SII	SIII	SV	SKXce	SLy4	SKT	RMFT [17]	Exp. [16]
^{116}Sn	4.656	4.678	4.603	4.622	4.634	4.610	4.609	4.625± 0.0019
^{118}Sn	4.672	4.696	4.617	4.638	4.649	4.627	4.623	4.639± 0.0019
^{119}Sn	4.681	4.705	4.623	4.647	4.657	4.636		4.643± 0.0020
^{120}Sn	4.687	4.714	4.630	4.655	4.664	4.645	4.636	4.651± 0.0021
^{122}Sn	4.699	4.725	4.639	4.664	4.674	4.653	4.649	4.663± 0.0022
^{124}Sn	4.710	4.735	4.648	4.674	4.684	4.662	4.661	4.673± 0.0023

Table-3: Calculated proton rms radii along with the RMFT results.

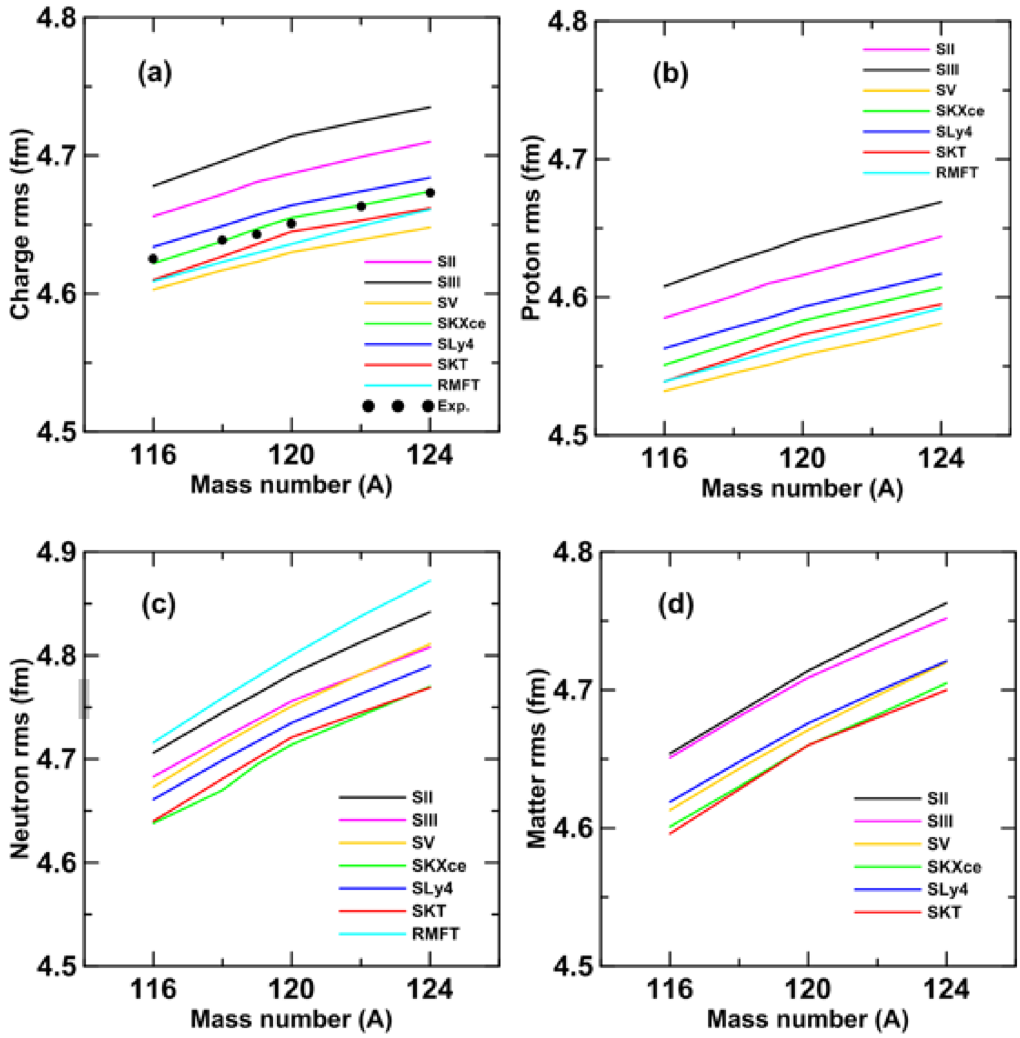
Nuclei	SII	SIII	SV	SKXce	SLy4	SKT	RMFT [17]
^{116}Sn	4.585	4.608	4.532	4.551	4.563	4.539	4.539
^{118}Sn	4.601	4.626	4.545	4.567	4.578	4.556	4.553
^{119}Sn	4.610	4.634	4.551	4.575	4.585	4.565	
^{120}Sn	4.616	4.643	4.558	4.583	4.593	4.573	4.567
^{122}Sn	4.630	4.656	4.569	4.595	4.605	4.584	4.579
^{124}Sn	4.644	4.669	4.581	4.607	4.617	4.595	4.592

Table- 4: Calculated neutron rms radii along with the RMFT results.

Nuclei	SII	SIII	SV	SKXce	SLy4	SKT	RMFT [17]	*t* (SLy4)
^{116}Sn	4.706	4.683	4.673	4.638	4.661	4.640	4.716	0.098
^{118}Sn	4.745	4.720	4.714	4.670	4.699	4.681	4.759	0.121
^{119}Sn	4.763	4.738	4.733	4.695	4.717	4.701		0.132
^{120}Sn	4.782	4.756	4.751	4.714	4.735	4.721	4.800	0.142
^{122}Sn	4.813	4.782	4.782	4.742	4.763	4.745	4.838	0.158
^{124}Sn	4.842	4.808	4.811	4.770	4.790	4.769	4.872	0.172

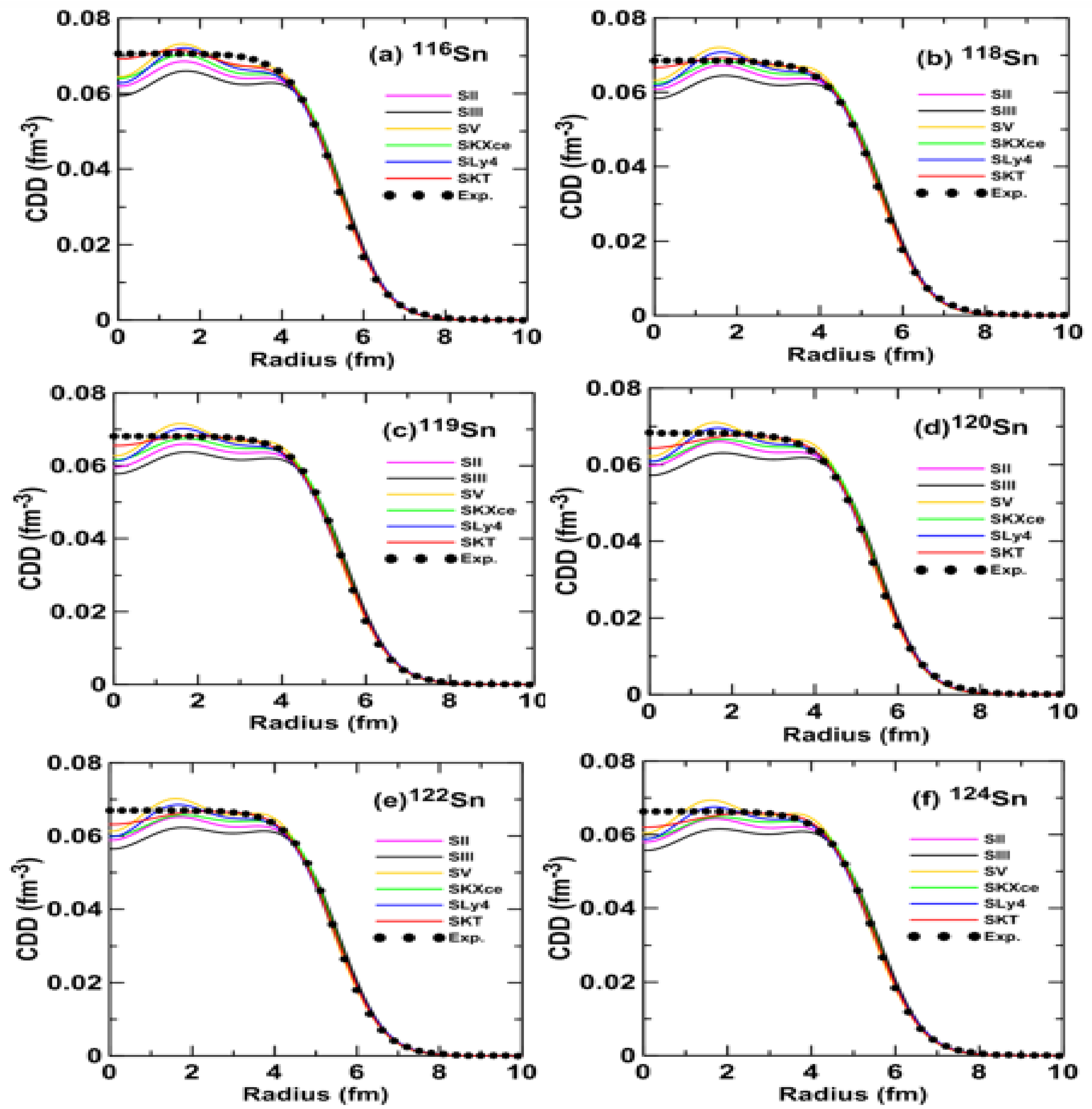
Table-5: Calculated matter rms radii.

Nuclei	SII	SIII	SV	SKXce	SLy4	SKT
^{116}Sn	4.654	4.651	4.613	4.601	4.619	4.596
^{118}Sn	4.684	4.681	4.643	4.630	4.648	4.628
^{119}Sn	4.699	4.695	4.657	4.645	4.662	4.644
^{120}Sn	4.714	4.709	4.671	4.660	4.676	4.660
^{122}Sn	4.739	4.731	4.696	4.682	4.699	4.680
^{124}Sn	4.763	4.752	4.720	4.705	4.721	4.700



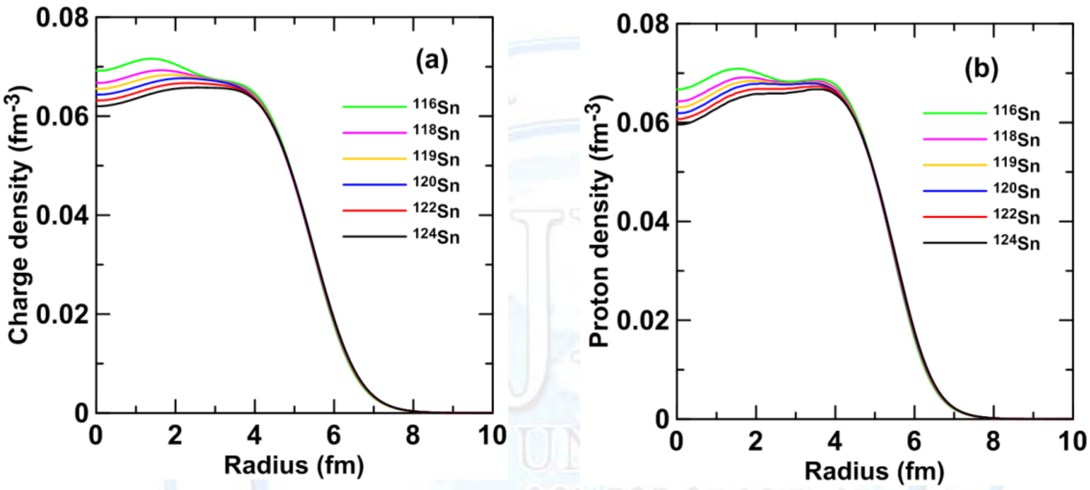
Figs.1a,b,c and d: The charge, proton, neutron and matter rms radii for Sn isotopes calculated with different Skyrme parameters.

The charge density distributions (CDD) of the selected isotopes are calculated using different Skyrme parameters and plotted in figures 2(a)-2(f). For comparison the experimental data [18,19] are also shown in these figures. It can be seen clearly from these figures that the best agreement between the experimental data and the calculated values is obtained by the SKT parameter where in this parameter the pairing correlations is carefully treated by the Bardeen-Cooper-Schrieffer (BCS) approximation and the effective mass is equal to the nucleon mass.



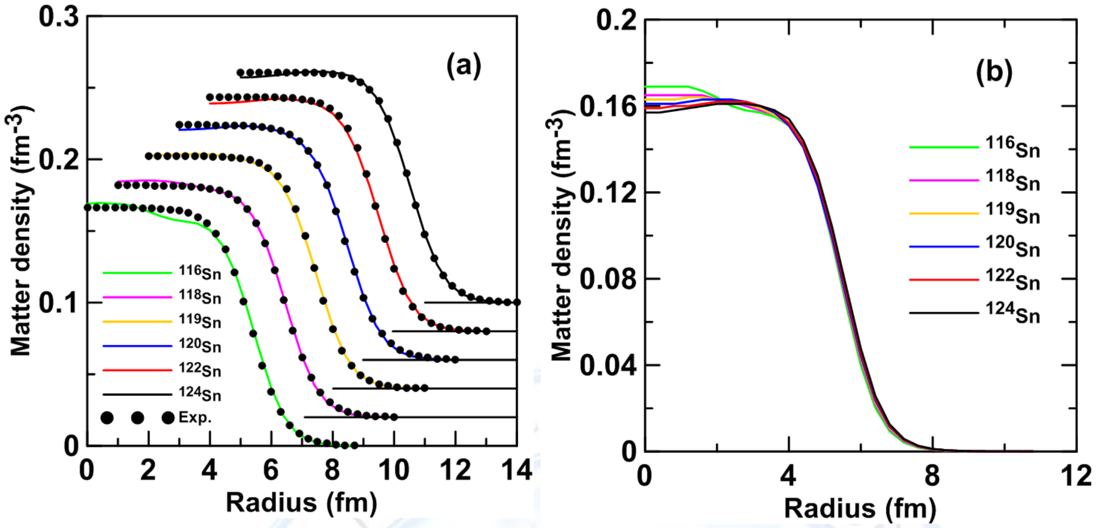
Figs.2a,b,c,d,e and f: The charge density distributions for Sn isotopes calculated by different Skyrme parameters along with measured data [18,19].

The charge and proton density distributions of the consider isotopes have been calculated using SHF with SKT parameterization and presented in figs. 3(a) and 3(b) as a function of *r* (fm). From these figures one can deduce that, at the center (r=0) the charge density is decreased approximately from (0.069 fm^{-3}) for ^{116}Sn to (0.062 fm^{-3}) for ^{124}Sn, while the proton density decreases approximately from (0.065 fm^{-3}) for ^{116}Sn to (0.06 fm^{-3}) for ^{124}Sn with increasing number of neutrons. The charge and proton densities are increase slowly as the radius increases up to about r = 2 fm and they are constant in the range from 2 fm to 4 fm, while they fall fairly rapidly to zero at the nuclear surface.



Figs.3 a and b: The density distributions for the Sn-isotopes calculated with SkT parameter; (a) charge and (b) proton.

The matter density distributions for Sn-isotopes have been also investigated and depicted in figs. 4(a) and 4(b). These profiles are calculated using SKT Skyrme parameter. The experimental matter densities of Sn-isotopes [18,19] are also shown in fig. 4(a) for comparison. As seen from this figure, the calculations by the SHF method reveal a good accordance with the measured data. Besides, the matter densities shown in fig. 4(b) are constant in the range of radii from 2 fm to 4 fm, while they decrease drastically to zero beyond (4 fm). Values close to zero are about in the vicinity of 9 fm to 10 fm.



Figs.4 a and b: Matter density distributions for Sn- isotopes calculated with SKT parameter . In fig. 4 (a) the curves and data have been progressively offset by 1 fm and 0.02 in the matter density. The filled circle symbols are the experimental data of refs. [18,19].

The theoretical binding energies per nucleon calculated using different Skyrme parameters for the isotopes under study are listed in table (6) and illustrate in fig. 5 compared with the corresponding experimental data [20] and RMFT [17] results. It might be noticed from this figure that the calculated results obtained by the SLy4 parameters are agree remarkably well with both the experimental and RMFT results.

Table-6: Calculated binding energy per nucleon along with experimental and RMFT results.

Nuclei	SII	SIII	SV	SKXce	SLy4	SKT	RMFT [17]	Exp. [20]
^{116}Sn	8.354	8.484	8.416	8.494	8.512	8.406	8.503	8.523
^{118}Sn	8.340	8.467	8.418	8.486	8.507	8.394	8.496	8.516
^{119}Sn	8.335	8.458	8.423	8.481	8.506	8.387	----	8.499
^{120}Sn	8.327	8.449	8.430	8.475	8.504	8.380	8.486	8.504
^{122}Sn	8.287	8.438	8.409	8.457	8.478	8.377	8.472	8.487
^{124}Sn	8.250	8.428	8.397	8.438	8.454	8.373	8.456	8.467

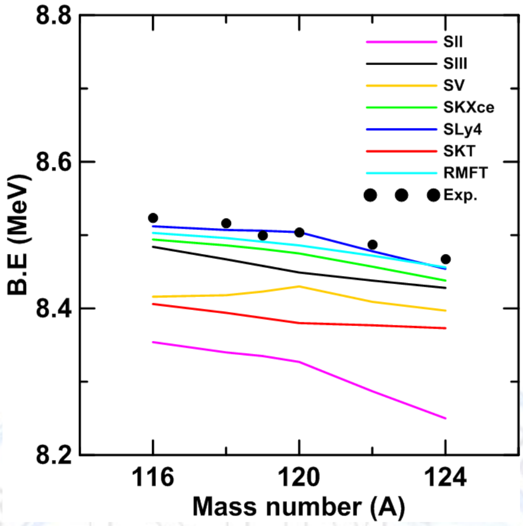
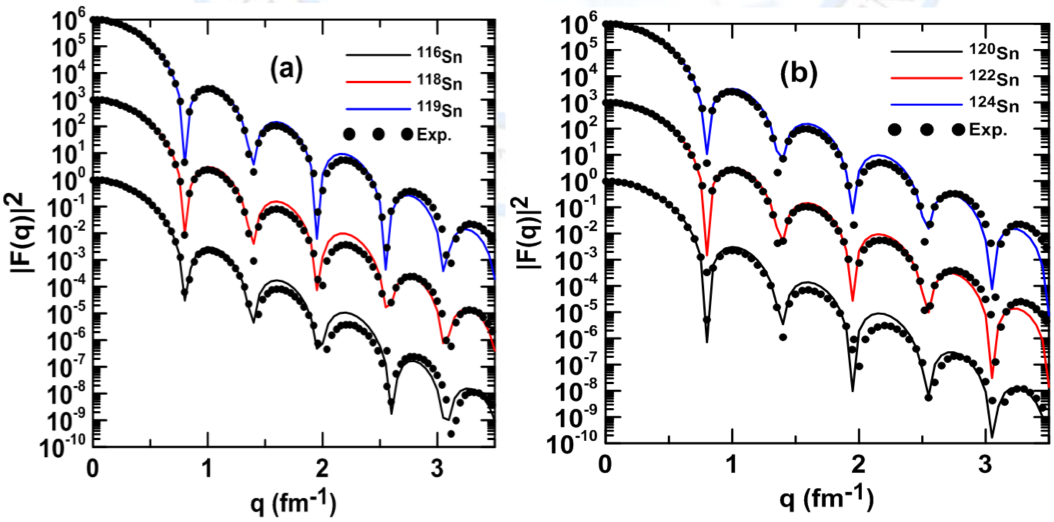


Fig. 5: Calculated binding energy per nucleon for Sn-isotopes compared those of RMFT [17] and experimental [20] results.

The calculated charge form factors with SKT parameter for isotopes under investigate compared with the experimental data (denoted by filled circle symbols) [18,19] are displayed in fig. 6. The experimental data are very well reproduced by calculated results throughout all range of momentum transfer (q).



Figs.6 a and b: Elastic charge form factors for Sn-isotopes obtained by SKT parameter . (a) ^{116}Sn, ^{118}Sn ($\times10^3$), ^{119}Sn ($\times10^6$), (b) ^{120}Sn, ^{122}Sn ($\times10^3$) and ^{124}Sn ($\times10^6$).

Conclusions

In this research, the charge, proton and matter densities, charge, proton, neutron and matter root mean square (rms) radii, neutron skin thickness, elastic charge form factor and the binding energy per nucleon for ^{116,118,119,120,122,124}Sn stable nuclei were calculated using the Skyrme-Hartree-Fock method. It is noted that the Skyrme-Hartree-Fock method is capable of reproducing information about the nuclear structures for both odd and even-A nuclei as do those of the experimental data. The results of this study may be considered as an indication to the validity of the SHF model for odd-A nuclei.

References

1. M. Beiner, H. Flocard, N. V. Giai and P. Quentin, *Nucl. Phys.*, A **238** (1975) 29.
2. L.G. Qiang, J. Phys. G. Nuclear and Particle Physics 17 (1991) 1.
3. E. Tel, Ş. Okuducu, G. Tanir, N. Akti , M. Bölükdemir, Communications in Theoretical Physics 49 (2008) 696.
4. T.W. Donnelly and I. Sick, Rev. Mod. Phys. 56 (1984) 461.
5. A.H. Wapstra, G. Audi, and R. Hoekstra, Atomic Data and Nuclear Data Tables 39 (1988) 281.
6. A. Huseyin, E. Tel and R. Baldik, Turk J. Phys. 32 (2008) 181.
7. Y. Shen and Z. Ren, Phys. Rev.C 54 (1996) 1158.
8. D. Vauthering and D.M. Brink, Phys. Rev. C 5 (1972) 626.
9. M. Beiner, H. Flocard, and N. V. Giai, Nucl. Phys. A238 (1975) 29.
10. B. A. Brown, Phys. Rev. C 58 (1998) 220.
11. E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and T. Schaeffer, Nucl. Phys. A 635 (1998) 231.
12. F. Tondeur, Phys. Lett. 123 B (1983) 139.
13. L.X. Ge, Y.Z. Zhuo, and W. Norenberg, Nucl. Phys. A 459 (1986) 77.
14. P. G. Reinhard, F. Hummer and K. Goeke, Z. Phys. A 317 (1984) 339.
15. C. A. Bertulani, J. Phys. G: Nucl. Part. Phys. 34 (2007) 315.
16. I. Angeli and K.P. Marinova, Atomic Data and Nuclear Data Tables 99 (2013) 69.

17. G A Lalazissis and S Raman, Atomic Data and Nuclear Data Tables 71 (1999) 1.

18. H. de Vries, C.W. de Jager, and C. de Vries, Atomic Data and Nuclear Data Tables 36 (1987) 495.

19. G. Fricke, C. Bernhardt, K. Heilig, L.A. Schaller, L. Shellenberg, E.B. Shera, and C.W. de Jager, Atomic Data and Nuclear Data Tables 60 (1995) 177.

20. M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. M. Cormick, X. Xu and B. Pfeiffer, Chinese Physics C 36 (2012) 1603.