A Theoretical Study of the Interaction between Supernovae and Their Surroundings

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Abstract

This work presents an overview of the physics of supernovae and of their interaction with a circumstellar medium. In particular the reverse shock created by the interaction is investigated. In most types of the supernovae this shock is radiative, and due to the high temperature most of the radiation comes out as X-rays.

We analyze the details of the numerical calculation of the adiabatic simulation to study the evolution of the earliest phases of a supernova explosion when the stellar atmosphere ejected by the explosion interacts with the environment of the star. This process is often called "circumstellar interaction".

The interaction depends on the properties of the wind, i.e. the wind velocity and mass loss rate of the progenitor as well as the composition of the wind.
We assumed that both the supernova ejecta and the environment are spherically symmetric, which allows us to solve the problem in one dimension, with as spatial variable the radius *r*.

Keywords: supernova: stars, Shock waves, X-ray

Introduction

In galaxies one finds a many sources of shock waves and the interstellar medium ISM is significantly influenced by the shocks. These shocks can arise from strong stellar winds and from stellar explosions, supernova.

Supernovae are caused by run-away thermonuclear reactions that occur when stellar cores collapse. In some type of supernovae such as type I involves a white dwarf that exceeds the Chandrasekhar mass limit on account of accretion from another star in close orbit. A type II supernova happens for massive stars when the iron core, for which no more energy gain by fusion is possible. Typical kinetic explosion energy is 10^{51} ergs or $\sim 10^{-3}M_{\odot}c^{2}$. In both cases the outer layers of the star are expelled or ejected with high velocity, which creates a strong shock when the ejecta meet the circumstellar medium. The latter may be dilute gas for a type I supernova or hot, magnetized stellar wind material for a type II supernova. [1]

The collision between the ejecta and the circumstellar medium creates two shocks; one moving outward into the circumstellar medium and a reverse shock that moves backward into the ejecta (Fig. 1). Because of the high temperatures involved, most of the emission from the interaction region is radiated as X-rays.

The supernova remnant is expands adiabatically. After some time of the expansion the mass swept up by the outwardly moving shock wave will significantly exceed the mass of the initial ejecta. The ram pressure, of the matter that enters the shock wave may be much larger than the thermal pressure of the upstream medium, Pu. When the radiative energy loss is much smaller than the initial available energy E at this stage, the supernova remnant is said to produce a blast wave of supernova SNR. [2]

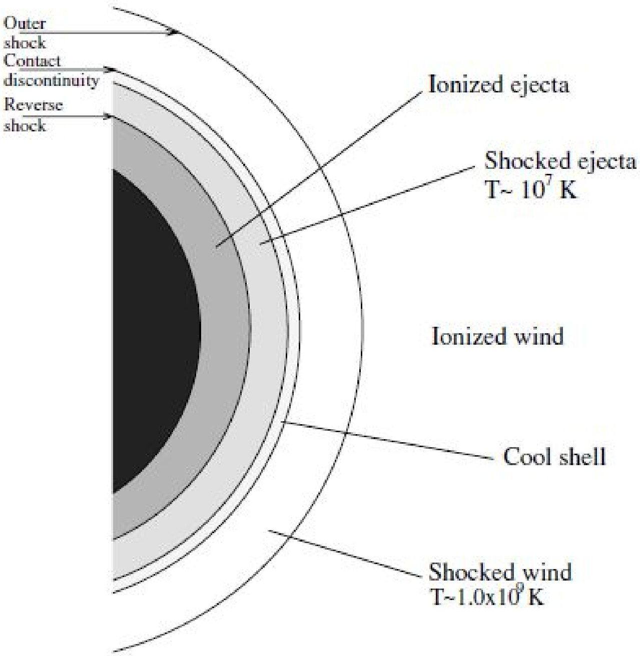


Figure 1: Schematic picture of the different regions in the interaction of supernova ejecta with the progenitor wind.

Theory

The blast wave of supernova will be spherically-symmetric evaluated in space r and time t. Neglecting the pressure of the external medium, P_u, for a moment, we have only the explosion energy, E, and the external density, ρ_u, as parameters of the problem. The hydrodynamical equations can be written in non-dimensional variables using scales, for example a radial scale r_0 for the radius coordinate. How could a dimensionless radius variable be composed, if only the radius r, time t, the energy E, and a density ρ_u are at our disposal? If we set

$$x = rt^{-2/5}\rho_o E^{-1/5} \qquad (1)$$

Where: x is a variable.

The time and location are coupled through the variable x. the flow at any location and any time is looking the same as it did at some other location and an earlier time. The flow is said to be self-similar.

The variation of shock radius r_s with time is

$$r_s(t) = x_o\left(\frac{Et^2}{\rho_o}\right)^{\frac{1}{5}} \tag{2}$$

x_0 is a fixed value.

The velocity of the shock wave then is

$$v_s(t) = \frac{dr_s}{dt} = \frac{2}{5}x_o\left(\frac{E}{\rho_o t^3}\right)^{\frac{1}{5}} \tag{3}$$

The blast wave thus decelerates and disappears after some time.

The supernova remnant turns from adiabatic expansion to a blast wave after the shock would have swept up a mass similar to the initial ejecta mass $(M = 0.5M_{\odot}\ x_o^5)$, if it was located at $x_0 \simeq 1$. [1]

The Sedov- Taylor solutions for a blast wave

It is a non-relativistic transformation to a frame that is fixed to the center of the remnant. So, there is the resulting self-similar solution for the flow.

Rankine-Hugoniont relations for the density, velocity, and pressure, for a strong shock are:

$$\rho = \left(\frac{\gamma+1}{\gamma-1}\right)\rho_o \tag{4}$$

$$v = \frac{2}{\gamma+1}v_s \tag{5}$$

$$P = \frac{2}{\gamma+1}\rho_o v_s^2 \tag{6}$$

$\gamma = 5/3$ is the ratio of the specific heats.

Applying the normal hydrodynamic equations. One can be obtained Taylor - Sedov solutions for the fluid quantities in self-similar form.

$$\rho(r,t) = \left(\frac{\gamma+1}{\gamma-1}\right)\rho_o\ \rho(\xi) \qquad (7)$$

$$v(r,t) = \frac{2}{\gamma+1} v_s\ v(\xi) \qquad (8)$$

$$P = \frac{2}{\gamma+1} \rho_o v_s^2\ p(\xi)$$

(9)

On the grounds of self-similarity, we have the dimensionless variables as a function of ξ alone, and the scaling here is such that $\rho(\xi) = v(\xi) = p(\xi)$=1. [3]

Implementation of the numerical simulation program.

We used a numerical simulation program of the gas dynamic and it is run on a Linux system by turn on the executable permission for the binary file.

This simulation calculates the evolution of the earliest phases of a supernova explosion. To run the simulation you will need to input parameters. The required input parameters are the size of the computational mesh (how many computational cells should be used), the size of the domain on which the calculation is done, some parameters describing the initial conditions, the time between outputs, and the total time for which to run the simulation (time here means physical time, not wall clock or CPU time).

The program will run until the maximum of output files have been generated.

For each output time a new file is created. The output files are ASCII tables containing four columns of the length of the mesh. The four columns are the radius (in cm), the mass density (ρ in g cm^{-3}), the velocity (u in cm s^{-1}), and the pressure (p in dyne cm^{-2} = g cm^{-1} s^{-2}).

The output files can be read into your favourite plotting program for further analysis.Our numerical results have been plotted using mathlab program.

Results of the numerical analysis and discussion

Our numerical results include two parts. In the first part we studied the changing in density and pressure profiles of an expanding supernova remnant. Then we satisfied the self-similar solutions of the flow. In the second part we studied the amount of X- ray emission from the interaction of the supernova ejecta and the environment.

Changing the density and pressure profile

The exploding star is surrounded by the remains of the stellar wind of the progenitor, the density of which is given by

$$\rho_w = \frac{\dot{M}}{4\pi R_*^2 v_w}\left(\frac{R_*}{r}\right)^s \qquad (10)$$

Where R_* is a reference radius, and the index *w* refers to the wind. A constant mass loss rate $\dot{M} = 4\pi r^2 \rho v$, and wind velocity v_w corresponds to the exponent s = 2, but the structure of the wind can be modified by successive periods of fast and slow winds, as well as by pulsation and binary interaction, all of which could contribute to changing the density gradient of the circumstellar medium.

The density structure of the outer ejecta of the exploding star can be approximated by a power law. [4]

$$\rho_{ej} = \rho_o \left(\frac{t}{t_o}\right)^{-3} \left(\frac{v_o t}{r}\right)^n \qquad (11)$$

Where ρ_o is the density at time t_o and velocity v_o. The density gradient *n* is usually in the range 7-12, but can be as large as ~ 20.

If the circumstellar medium is dense, the ram pressure which it exerts on the outgoing shock creates another shock (the reverse shock) which is driven backwards into the ejected material. While the reverse shock is travelling backwards in mass, both shocks usually travel outward in radius with the expansion velocity of the ejecta. [1]

We run the simulation for a number of grids of 4000 of a domain size 2.5 x 10^{15}cm, and we used the following input parameters:

$n = 9$

$r_o = 5 \times 10^{14}$ cm

$\rho_o = 1.3 \times 10^{-18}$ g cm $^{-3}$

$v_o = 5.8 \times 10^4$ km s^{-1} and $v_{wind} = 10$ km s^{-1}

$$\dot{M}_{wind} = 5 \, x \, 10^{-5} M_{\odot} yr^{-1}$$

We take 0.1 days for the output times, and for the total times 30 days.

The programs will run until the generated flow patterns have reached an edge of the grid of 4000. A solution for ρ, and P of an expanding supernova remnant is shown in figure 2. The variations across the grid for these quantities are such that it is better to plot the logarithm ($\log_{10}$) of these quantities.

So, the density and pressure plotted in $\log_{10}$ – scale as a function of radius. In this figure we see the following regions from left to right: freely expanding supernova ejecta, an inner shock, shocked supernova ejecta, contact discontinuity, shock environment, outer shock and undisturbed environment.

It is clear also that the interaction between the supernova ejecta and the environment of the star creates the forward shock which is propagating through the environment and the reverse shock which is propagating through the ejecta, the two shocks separated by a contact discontinuity.

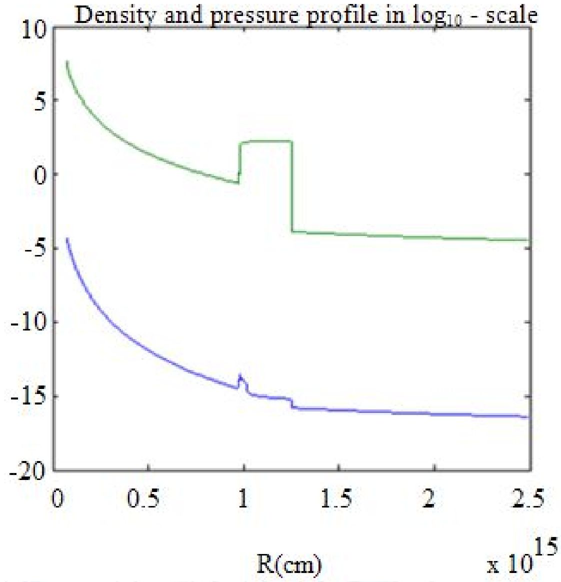


Figure 2: Density and pressure profiles as a function of radius. Pressure profile in the upper part of the figure, and density profile in the lower part of the figure.

During the early stages of the evolution, both the forward and reverse shocks expand outwards in radius.

The decelerating contact discontinuity is unstable, and shocked ejecta material will mix with the shocked ambient medium. At a later time, depending on the ejecta profile, the ejecta density becomes low enough that the reverse shock will “turn over” and actually begin to move inwards in radius, towards the explosion center.

To verify the self-similar solution for the flow, we need to study shock evolution at various times of the expansion. In this case we plotted shock evolution at various times of the simulation, and we dependent the changing density profile for 5,9, and 15days because the blast wave begin to decelerate and disappear after that time as we will see in X-ray emission in the second part.

Figure 3 shows the shock evolution at various times of the simulation. Here we need to calculate the ratios of the inner and outer shock to the contact discontinuities for the 5, 9, 15 days.

For ex. In figure 3, values of the inner shock R_1, contact discontinuity R_c, outer shock R_2, for the 9 day are equal to 1.25, 1.025, 0.95 respectively. By the same way one can calculate these values for the other two days. Using these values we measured the ratio of the ratios of the inner and outer shock to the contact discontinuities to satisfy that these results match the self-similar solution for all times. The results are clear in table 1.

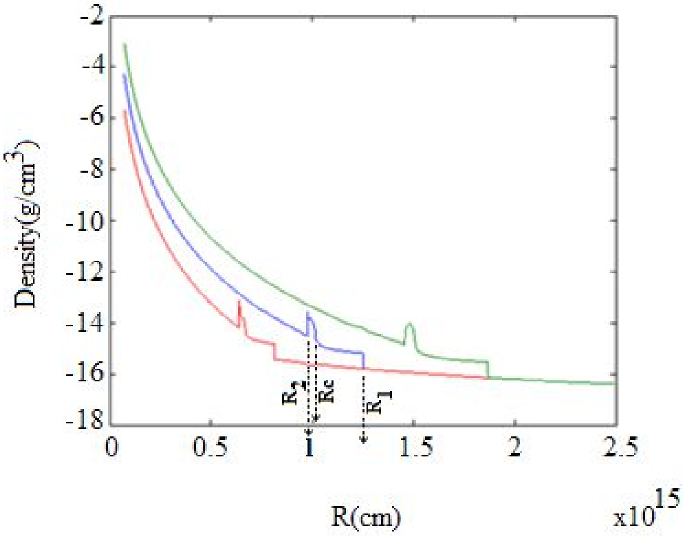
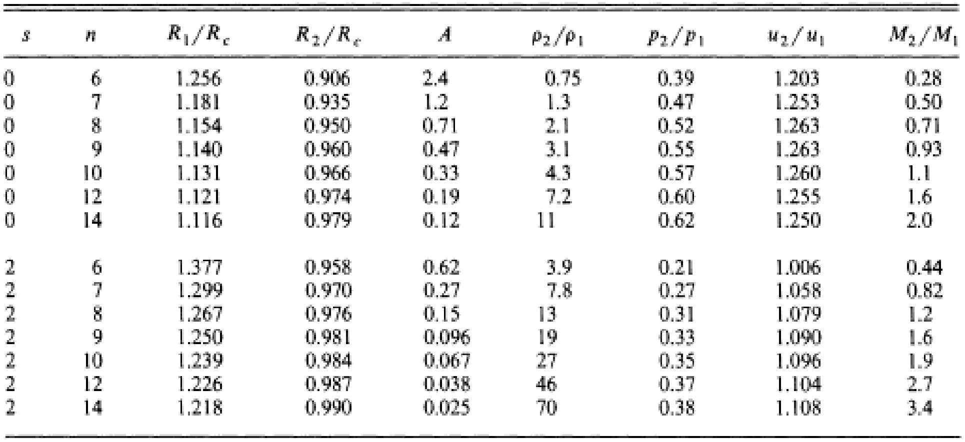


Figure 3: Density profiles for 5, 9, 15 days from the left to the right respectively, as a function of radius.

(R_1/Rc, R_2/Rc)	**(1.22,1.07)**	**(1.23,0.962)**	**(1.23,0.963)**
Days	5 days	9 days	15 days

Table 1: Ratios of the inner and outer shock position to the contact discontinuity for 5, 9, 15 days of the simulation.

If we compare our results with the full set of parameters for the self-similar solutions given in the table 2 for the case of s = 2, n = 9. We conclude that these results are matching the self-similar solution for all times of the simulation.



s	n	R_1/R_c	R_2/R_c	A	ρ_2/ρ_1	p_2/p_1	u_2/u_1	M_2/M_1
0	6	1.256	0.906	2.4	0.75	0.39	1.203	0.28
0	7	1.181	0.935	1.2	1.3	0.47	1.253	0.50
0	8	1.154	0.950	0.71	2.1	0.52	1.263	0.71
0	9	1.140	0.960	0.47	3.1	0.55	1.263	0.93
0	10	1.131	0.966	0.33	4.3	0.57	1.260	1.1
0	12	1.121	0.974	0.19	7.2	0.60	1.255	1.6
0	14	1.116	0.979	0.12	11	0.62	1.250	2.0
2	6	1.377	0.958	0.62	3.9	0.21	1.006	0.44
2	7	1.299	0.970	0.27	7.8	0.27	1.058	0.82
2	8	1.267	0.976	0.15	13	0.31	1.079	1.2
2	9	1.250	0.981	0.096	19	0.33	1.090	1.6
2	10	1.239	0.984	0.067	27	0.35	1.096	1.9
2	12	1.226	0.987	0.038	46	0.37	1.104	2.7
2	14	1.218	0.990	0.025	70	0.38	1.108	3.4

Table 2: Properties of the self-similar solutions. The table is taking from reference [5]

X-ray emission of the supernova

In this part we calculated the X-ray emissivity and luminosity of the supernova using Mathlab program as follows.

X-ray emissivity of the supernova

The output file for the values of the pressure, P, and density, ρ, which we obtained from the simulation, can be used as input file in Mathlab program to calculate the x-ray emissivity as follow.

The x-ray emissivity of a hot gas at photon energy $h\nu$ of 100 keV can be approximated by (free-free emission from hydrogen[2]) as: [6]

$$4\pi n^2 \frac{f_k}{\sqrt{T}} \exp\left(-\frac{h\nu}{K_B T}\right) \qquad erg\, s^{-1} Hz^{-1} \qquad (12)$$

With f_k =5.44436 x 10^{-39}

K_B is Boltzmann constant.

n is the number density (of protons and electrons respectively) and it is equal to $(n = \rho / m_H)$.

m_H is mass of the hydrogen atom .

We need also to calculate the temperature T as: $T = \mu \frac{m_H}{K_B} \frac{P}{\rho}$

$\mu = 0.6$ is the mean molecular weight.

Using the above values we calculated the X-ray emissivity of the supernova and plot it as a function of radius in figure 4. This figure shows the X-ray emission from the interaction of the supernova ejecta and the environment. It is clear that both shocked regions emit x-rays, with a maximum emission in the inner shock region. The maximum emission peak of the x-ray in the inner Shock region ~ 9.2 x 10^{-8} erg s^{-1} cm^{-3} at radius ≈ $1x10^{15}$ cm and a minimum x-ray emission in the outer shock ~ 3 x 10^{3} erg s^{-1} cm^{-3} at radius ≈ $1.25x10^{15}$ cm.

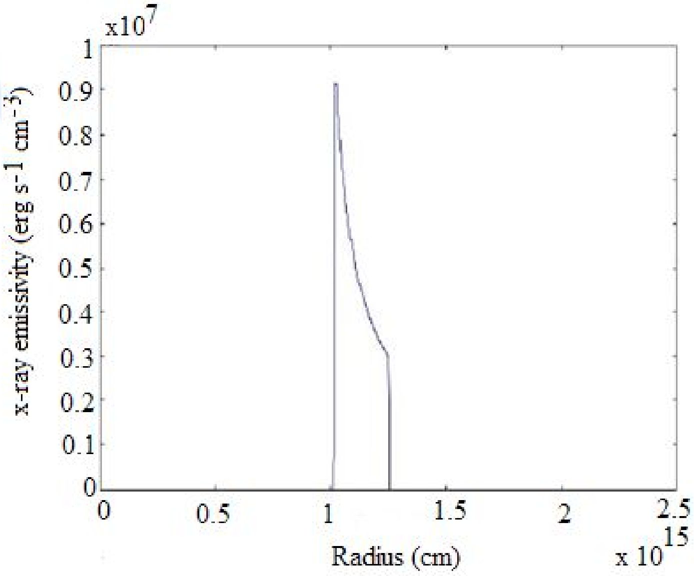


Figure 4: X-ray emissivity versus radius.

X-ray luminosity of the supernova

To study the light curve of the supernova we need to plot the luminosity versus time.

The x-ray luminosity is, [6]

$$4\pi n^2 \frac{f_k}{\sqrt{T}} \exp\left(-\frac{hv}{K_B T}\right) \; 2\pi r^2 \Delta r \;\; v \qquad (13)$$

The volume of each cell is $4\pi r^2 \Delta r$.

v is a bandwidth of 1 keV.

Multiplying values of the X-ray emissivity which we calculated in the previous section by the volume of each cell and a bandwidth of 1 Kev we obtained the X-ray luminosity of the supernova during 22 days.

The total x-ray luminosity of the supernova in units of solar luminous has been calculated by summing over all radial points and the results were plotted as a function of time to represent the light curve of the supernova in figure 5.

We see that the light curve in the figure starts with a sharp rise in luminosity, and then reaches a maximum luminosity ~ $2.75\, x\, 10^5 L_\odot$ over ~ one day, and then the light curve starts to decrease gradually to produce the declining part of the light curve after 10 days.

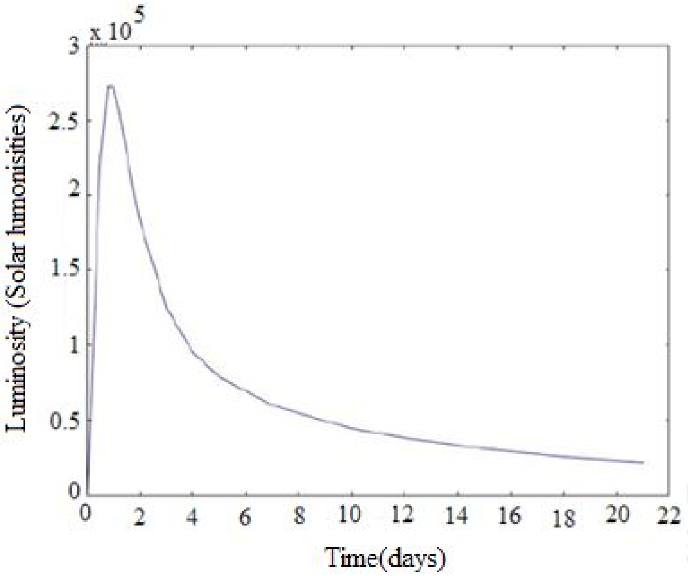


Figure 5: The total x – ray luminosity of the supernova as a function of time.

Conclusion

1. Blast waves from SNR expand into ISM / CSM leaving behind a supernova remnant. The first stages of the expansion can be described self-similarly and consist of a front shock and a reverse shock.

2. X-ray emission in the first days comes mainly from the shocked wind the cool shell is transparent. At later times the X-ray luminosity of the outer shock is too low and the reverse shock comes to dominate the X-ray emission.

3. Light curves display the temporal evolution of the energy output of a SNR. It decays linearly after the maximum, with a steep decline during the first few days after maximum emission, and the emission stays constant for a month.

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A Theoretical Study of the Interaction between Supernovae and Their Surroundings

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دراسة نظرية لتفاعلات المستعرات العظمى مع المحيط النجمي

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الخلاصة

في هذا العمل تم تقديم لمحة عامة عن فيزياء المستعرات العظمى (supernova) و تفاعلها مع المحيط النجمي. حيث تم التحقق عمليا من توليد موجات تصادم معكوسة (Shock waves) في هذه المستعرات. كما ان هذه الموجات تكون مشعة في معظم انواع المستعرات و بسبب درجات الحرارة العالية فان معظم هذه الاشعة تنبعث على شكل اشعة سينية (X-ray).

لقد تم تحليل الحسابات عدديا لدراسة تطور المراحل المبكرة من انفجار المستعرات العظمى عندما يتم قذف الغلاف الجوي النجمي بسبب التفاعل مع محيط النجم. و تسمى هذه الطريقة (circumstellar). كما ان التفاعل يعتمد على خصائص الرياح، سرعة الرياح، و معدل الكتلة المفقودة للمستعرات العظمى الاسلاف و بالاضافة الى تشكيل الرياح.

لتسهيل حل المشكلة في بعد واحد تم فرض ان كل من المستعرات العظمى المنقذفة و المحيط متماثلة كرويا.

كلمات مفتاحية: المستعرات العظمى: النجوم ، الموجات المتصادمة، الاشعة السينية