New Results of Fuzzy Continuous Functions on Fuzzy Topological spaces

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Abstract

In this paper we give further results about concerning certain to fuzzy continuous functions defined from fuzzy topological space to another fuzzy topological space and to show the relationships between fuzzy continuous functions where we confine our study to some of their types such as, fuzzy θ-continuous function , fuzzy strong θ-continuous and fuzzy δ-continuous function.

Keywords: fuzzy topological space , fuzzy δ-closure , fuzzy θ- closure , fuzzy θ-continuous function ,fuzzy strong θ-continuous ,fuzzy δ-continuous function.

نتائج جديدة للدوال المستمرة الضبابية على الفضاءات التبولوجية الضبابي

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الخلاصـــــــة

في هذا البحث نعطي نتائج اخرى حول بعض الانواع للدوال المستمرة الضبابية من فضاء التوبولوجي ضبابي الى فضاء تبولوجي اخر واظهار العلاقات بين الدوال المستمرة الضبابية حيث تنحصر دراستنا حول بعض الانواع مثل fuzzy θ - continuous function ,fuzzy strong θ -continuous , fuzzy δ -continuous function.

الكلمـــات الدليلية الفضاء التبولوجي الضبابي, fuzzy θ–closure , fuzzy θ-continuous , fuzzy δ-closure , fuzzy function ,fuzzy strong θ-continuous , fuzzy δ-continuous function.

Introduction

The study of fuzzy sets was initiated with the famous paper of Zadeh [12] in 1965, and thereafter the paper of Chang [3] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts .The concepts of fuzzy δ-closure and fuzzy θ -closure in fuzzy topological space were introduced by Ganguly and saha [10] and Mukherjee and sinha [6] respectively. In this paper we investigate some properties fuzzy strong θ - continuous, fuzzy θ -continuous and fuzzy δ-continuous .

Preliminaries

The definitions and results which are used in this paper concerning fuzzy topological spaces have already taken certain standard shape a fuzzy set Ã in a fuzzy topological space (X, Ť) (fts X, for short) is said to be quasi-coincident (q- coincident, for short) with a fuzzy set B̃, denoted by ÃqB̃ if and only if there exists x ∈ X such that $\mu_{\tilde{A}}(x)+\mu_{\tilde{B}}(x) > 1$ [4]. A fuzzy set in (X,Ť) is said to be quasi-neighborhood (q-nbd , for short) of a fuzzy point X_r or p̃ (where X is the support and r is the value of the fuzzy point $0 < r \leq 1$) if and only if there exists a fuzzy open set B̃ such that p̃ q B̃ ≤ Ã [4] .For two fuzzy sets Ã , B̃ in X, Ã ⊆ B̃ iff Ã is not q-coincident with $\tilde{B}^c$ (complement of B̃) to be denoted by $\tilde{A} \not{q} \tilde{B}^c$. By cl(Ã) and int(Ã) we shall denote respectively the fuzzy closure and fuzzy interior of a fuzzy set Ã in a fts X. A fuzzy point p̃ ∈ cl(Ã) if and only if each q-nbd of p̃ is q-coincident with Ã [4].

Definition 2.1 [4]. A fuzzy set $\tilde{A}$ of a fuzzy topology space $(X,\tilde{T})$ is called

(a) A fuzzy regular closed set of X, if $cl(int(\tilde{A})) = \tilde{A}$

(b) A fuzzy regular open set of X, if $int(cl(\tilde{A})) = \tilde{A}$.

Remark 2.2 [2],[7]. Every fuzzy regular open set is a fuzzy open set and every fuzzy regular closed set is a fuzzy closed set.

Definition 2.3 [11]. A fuzzy point $\tilde{p}$ in (X, $\tilde{T}$) is said to be a fuzzy (δ-cluster point) of a fuzzy set $\tilde{A}$ if for each fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$, int(cl($\tilde{U}$)) q $\tilde{A}$.

The set of all fuzzy (δ-cluster points) of $\tilde{A}$ is called the fuzzy (δ-closure) of $\tilde{A}$ and is denoted by

$[\tilde{A}]_\delta$. A fuzzy set $\tilde{A}$ is fuzzy (δ-closed) if and only if ($\tilde{A} = [\tilde{A}]_\delta$) and the complement of a fuzzy set (δ-closed set) is fuzzy (δ-open) set.

Remark 2.4 [1],[5] [6]. Every fuzzy regular open set is a fuzzy δ-open set and a fuzzy δ-open set is

a union of fuzzy regular open sets and hence is a fuzzy open set and for a fuzzy open set $\tilde{A}$ in a fts

(X, $\tilde{T}$), cl($\tilde{A}$) = $[\tilde{A}]_\delta = [\tilde{A}]_\theta$.

Definition 2.5 [6]. A fuzzy point $\tilde{p}$ is said to be fuzzy θ-cluster point of a fuzzy set $\tilde{A}$ if and only if for every open q-nbd. $\tilde{U}$ of $\tilde{p}$, cl($\tilde{U}$) is quasi-coincident with $\tilde{A}$. The set of all fuzzy θ-cluster points of $\tilde{A}$ is called the fuzzy θ-closure of $\tilde{A}$ and will be denoted by $[\tilde{A}]_\theta$.

A fuzzy set $\tilde{A}$ will be called fuzzy θ-closed if and only if $\tilde{A} = [\tilde{A}]_\theta$ and the complement of fuzzy θ-closed set is a fuzzy θ-open.

Definition 2.6 [4]. Let (X,Ť) be a fts , and A is any subset of X ; then the family $Ť_A$, defined by $Ť_A$= { B̃ / A : B̃ ϵ Ť }, which is obviously a fuzzy topology for A ,is called relative fuzzy topology, or the relativization of Ť to A. Such a fuzzy topological space (A,$Ť_A$) is called a subspace of (X,Ť) .

Characterizations of fuzzy $\theta-continuity$, fuzzy strong $\theta-continuity$ and fuzzy δ-continuity.

Definition 3.1 [8]. A function $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$, is said to be

(a) Fuzzy θ-continuous (f.θ.c, for short) if for each fuzzy point $\tilde{p}$ in $(X, \tilde{T})$ and each fuzzy open q-nbd. $\tilde{V}$ of $f(\tilde{p})$, there exists fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$ such that $f(cl(\tilde{U})) \subseteq cl(\tilde{V})$.

(b) Fuzzy strong θ-continuous (f.s.θ.c, for short) if for each fuzzy point $\tilde{p}$ in $(X, \tilde{T})$ and each fuzzy open q-nbd. $\tilde{V}$ of $f(\tilde{p})$, there exists fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$ such that $f(cl(\tilde{U})) \subseteq \tilde{V}$.

Definition 3.2 [9]. A function $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$, is said to be fuzzy δ-continuous at a fuzzy point $\tilde{p}$ in $(X, \tilde{T})$, if for each fuzzy open q-nbd. $\tilde{V}$ of $f(\tilde{p})$, there exists an open fuzzy nbd. $\tilde{U}$ of $\tilde{p}$ such that $f(int(cl(\tilde{U})) \subseteq int(cl(\tilde{V}))$.

Also, the next theorem appears in [6], [8] with uncompleted proof and hence we give the complete details of the proof.

Theorem 3.3. For a function $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$, each of the following statements equivalent:

(a) f is fuzzy θ-continuous function.

(b) For each fuzzy set $\tilde{A}$ in X, $f([\tilde{A}]_\theta) \subseteq [f(\tilde{A})]_\theta$.

(c) For each fuzzy set $\tilde{B}$ in Y, $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}([\tilde{B}]_\theta)$.

(d) For each θ-closed set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is a fuzzy θ-closed in X.

(e) For each θ-open set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is a fuzzy θ-open in X.

(F) For each open set $\tilde{B}$ in Y, $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(cl(\tilde{B}))$.

Proof:

(a)$\Rightarrow$(b).Let $\tilde{p}$ be a fuzzy point $\in [\tilde{A}]_\theta$ and let $\tilde{U}$ any open q-nbd. of f($\tilde{p}$). Then there exists an open q-nbd. $\tilde{V}$ of $\tilde{p}$ such that f(cl($\tilde{V}$)) $\subseteq$ cl($\tilde{U}$), i.e., f is a fuzzy θ-continuous.

Now $\tilde{p} \in [\tilde{A}]_\theta$, cl($\tilde{V}$) q $\tilde{A}$ $\Rightarrow$ f(cl($\tilde{V}$)) q f($\tilde{A}$)

cl$\tilde{U}$ q f($\tilde{A}$) $\Rightarrow$f($\tilde{p}$) $\in$ [f($\tilde{A}$)] $\Rightarrow$ $\tilde{p} \in f^{-1}([f(\tilde{A})]_\theta)$.

Thus, $[\tilde{A}]_\theta \subseteq f^{-1}([\tilde{A}]_\theta)$ so that $f([\tilde{A}]_\theta) \subseteq [f(\tilde{A})]_\theta$

(b) $\Rightarrow$ (c). By (b) $f([f^{-1}(\tilde{B})]_\theta) \subseteq [ff^{-1}(\tilde{B})]_\theta \subseteq [\tilde{B}]_\theta$.

Which implies that $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}([\tilde{B}]_\theta)$.

(c) $\Rightarrow$ (d). We have $[\tilde{B}]_\theta = \tilde{B}$

Now by (c)$\Rightarrow$ $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}([\tilde{B}]_\theta) = f^{-1}(\tilde{B})$.$\Rightarrow$$[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(\tilde{B})$

For each fuzzy θ-closed set $\tilde{B}$ in Y. $f^{-1}(\tilde{B})$ is a fuzzy θ-closed in X.

(e)$\Rightarrow$ (f). Since $\tilde{B}$ be a fuzzy open in Y, by remark (2.4) cl($\tilde{B}$) = $[\tilde{B}]_\theta$ and we have from (c) $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}([\tilde{B}]_\theta)$. We have $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(cl(\tilde{B}))$

(f)$\Rightarrow$(a). Let $\tilde{p}$ be a fuzzy point in X and $\tilde{V}$ be a fuzzy open q-nbd. of f($\tilde{p}$). Then $\mu_{(1-cl(\tilde{V}))}(x)$ is a fuzzy open in Y. By (f), we have $[f^{-1}(1-cl(\tilde{V}))]_\theta \subseteq f^{-1}(cl(1-cl(\tilde{V})))$

$$= 1 - f^{-1}(int(cl(\tilde{V}))).$$

Then there exists a fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$, such that cl($\tilde{U}$) $\not{q}$ $(1 - f^{-1}(cl(\tilde{V})))$ so that f(cl($\tilde{U}$)) $\subseteq$ cl($\tilde{V}$).Hence, f is a fuzzy θ-continuous function.

Both theorems appear in [8], [5] respectively, without a complete proof. We presented the proof completely for its importance as we had done in the previous theorems.

Theorem 3.4. For a function $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$, the following are equivalent:

(a) f is fuzzy strong θ-continuous.

(b) $f([\tilde{A}]_\theta) \subseteq cl(f(\tilde{A}))$ for every fuzzy set $\tilde{A}$ in X,.

(c) $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(cl(\tilde{B}))$ for every fuzzy set $\tilde{B}$ in Y.

(d) The inverse image of every fuzzy closed set in Y is fuzzy θ-closed in X, i.e., every fuzzy closed set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is fuzzy θ-closed in X.

(e) The inverse image of every fuzzy open set in Y is fuzzy θ-open in X, i.e., every fuzzy open set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is a fuzzy θ-open in X.

Proof:

(a) ⇒ (b). Let $\tilde{p} \in [\tilde{A}]_\theta$ and $\tilde{V}$ be fuzzy open q-nbd. of $f(\tilde{p})$.

By (a), there exists a fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$ such that $f(cl(\tilde{U})) \subseteq \tilde{V}$ .Now, we have $\tilde{p} \in [\tilde{A}]_\theta$

$\Rightarrow cl(\tilde{U})\ q\ \tilde{A} \Rightarrow f(cl(\tilde{U}))\ q\ f(\tilde{A}) \Rightarrow \tilde{V}\ q\ f(\tilde{A}) \Rightarrow f(\tilde{p}) \in cl(f(\tilde{A})) \Rightarrow \tilde{p} \in f^{-1}(cl(f(\tilde{A})))$

Hence, $[\tilde{A}]_\theta \subseteq f^{-1}(cl(f(\tilde{A})))$ and so $f([\tilde{A}]_\theta) \subseteq cl(f(\tilde{A}))$.

(b) ⇒ (c). Let $\tilde{B}$ be a fuzzy set in Y.By (b) $f([f^{-1}(\tilde{B})]_\theta) \subseteq cl(f(f^{-1}(\tilde{B}))).\Rightarrow f([f^{-1}(\tilde{B})]_\theta) \subseteq cl(\tilde{B})$

$\Rightarrow f^{-1}f([f^{-1}(\tilde{B})]_\theta) \subseteq f^{-1}(cl(\tilde{B}))\Rightarrow [f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(cl(\tilde{B}))$

(c) ⇒ (d). Let $\tilde{B}$ be a fuzzy closed in Y.

By (c), we have $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(cl(\tilde{B})) = f^{-1}(\tilde{B})$ (since $\tilde{B}$ closed), which implies that $f^{-1}(\tilde{B}) = [f^{-1}(\tilde{B})]_\theta$.Hence, $f^{-1}(\tilde{B})$ is fuzzy θ-closed (c)⇒ (e). Let $\tilde{B}$ be a fuzzy open in Y. Then $\mu_{(1-\tilde{B})}(x)$ is a fuzzy closed and by (d),

$f^{-1}(1 - \tilde{B}) = 1 - f^{-1}(\tilde{B})$ is fuzzy θ-closed. Hence, $f^{-1}(\tilde{B})$ is fuzzy θ-open.

(e)⇒(a). Let $\tilde{p}$ be any fuzzy point in X and $\tilde{V}$ be a fuzzy open q-nbd. of $f(\tilde{p})$. By (e), $f^{-1}(\tilde{V})$ is a fuzzy θ-open in X.

Now, $f(\tilde{p})\ q\ \tilde{V} \Rightarrow \tilde{p}\ q\ f^{-1}(\tilde{V}) \Rightarrow \tilde{p} \notin 1 - f^{-1}(\tilde{V})$

Hence $\mu_{(1-f^{-1}(\tilde{V}))}(x)$ is fuzzy θ-closed set such that $\tilde{p} \notin 1 - f^{-1}(\tilde{V})$.

Then, there exists fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$, such that $cl(\tilde{U})\ \not{q}\ (1 - f^{-1}(\tilde{V}))$, which implies $f(cl(\tilde{U})) \subseteq \tilde{V}$. This shows that f is fuzzy strong θ-continuous function.

Theorem 3.5. Let $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$, then the following implications holds:

(a) f is a fuzzy δ-continuous function.

(b) $f([\tilde{A}]_\delta) \subseteq [f(\tilde{A})]_\delta$ for every fuzzy set $\tilde{A}$ in X.

(c) $[f^{-1}(\tilde{B})]_\delta \subseteq f^{-1}([\tilde{B}]_\delta)$ for every fuzzy set $\tilde{B}$ in Y.

(d) For every fuzzy δ-closed set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is a fuzzy δ-closed in X.

(e) For every fuzzy δ-open set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is fuzzy δ-open in X.

(f) For every fuzzy regular open set $\tilde{B}$ in Y, $f^{-1}(\tilde{B})$ is δ-open in X.

Proof:

(a) ⇒ (b). We show that for a subset $\tilde{A}$ of X, $f([\tilde{A}]_\delta) \subseteq [f(\tilde{A})]_\delta$

Let $\tilde{p}$ be a fuzzy point in Y such that $\tilde{p} \in f([\tilde{A}]_\delta)$. Then $\tilde{p} = f(\tilde{q})$, where $\tilde{q} \in [\tilde{A}]_\delta$.

It may be seen that if for a fuzzy subset $\tilde{A}$ of X, $\tilde{p} \in f(\tilde{A})$ then there is an $x \in$ supp $\tilde{A}$ such that $f(x) = y$; Let $\tilde{U}$ be a regularly open q-nbd. of $\tilde{p}$; there exists a regularly open q-nbd. $\tilde{V}$ of $\tilde{q}$ such that $f(\tilde{V}) \subseteq \tilde{U}$. Now, $\tilde{q} \in [\tilde{A}]_\delta$, which implies to $\tilde{V}\, q\, \tilde{A}$.

If possible $f(\tilde{V}) \not{q}\, f(\tilde{A})$. Then for all $y \in \tilde{V}$, $f(\tilde{V})(y) + f(\tilde{A})(y) \leq 1$, i.e., $\tilde{V} \not{q}\, \tilde{A} \Rightarrow$ a contradiction

Thus, $f(\tilde{V})\, q\, f(\tilde{A})$, i.e., $\tilde{U}\, q\, f(\tilde{A})$, i.e., $\tilde{p} \in [f(\tilde{A})]_\delta$, i.e., $f([\tilde{A}]_\delta) \subseteq [f(\tilde{A})]_\delta$.

(b) $\Rightarrow$ (c). By (b) $f([f^{-1}(\tilde{B})]_\delta) \subseteq [f(f^{-1}(\tilde{B}))]_\delta$, so that: $f([f^{-1}(\tilde{B})]_\delta) \subseteq [\tilde{B}]_\delta$ i.e.$[f^{-1}(\tilde{B})]_\delta \subseteq f^{-1}([\tilde{B}]_\delta)$.

(c) $\Rightarrow$ (d). Let $\tilde{B}$ be a fuzzy δ-closed set in Y. Then $[\tilde{B}]_\delta = \tilde{B}$, by (c)$[f^{-1}(\tilde{B})]_\delta \subseteq f^{-1}([\tilde{B}]_\delta) = f^{-1}(\tilde{B})$.

Thus, $f^{-1}(\tilde{B})$ is a fuzzy δ-closed in X.

(d) $\Rightarrow$ (e). Let $\tilde{B}$ be a fuzzy δ-open set in Y.

Then $\tilde{B}^c$ is a fuzzy δ-closed in Y. By (d), $f^{-1}(\tilde{B})$ is a fuzzy δ-closed in X. but $f^{-1}(\tilde{B}^c) = (f^{-1}(\tilde{B}))^c$

$f^{-1}(\tilde{B})$ is a fuzzy δ-open in X.

(e) $\Rightarrow$ (f). Every regular open fuzzy set is δ-open fuzzy set also. Thus (e) $\Rightarrow$ (f)

(f) $\Rightarrow$ (a). Let $\tilde{B}$ be a regular open q-nbd. of $f(\tilde{p})$ such that $\tilde{p}$ be a fuzzy point in X. by (f) $f^{-1}(\tilde{B})$ is

δ-open in X and $\tilde{p}\, q\, f^{-1}(\tilde{B})$. But every a δ-open fuzzy set is a union of fuzzy regular open sets [Remark (2.4)] and hence is an open fuzzy set.

Hence, there exists a fuzzy regular open sets $\tilde{A}$ such that $\tilde{p} \in \tilde{A} \subseteq f^{-1}(\tilde{B})$.

Now, $f(\tilde{A}) \subseteq \tilde{B}$ so $f(Int(cl(\tilde{A}))) \subseteq Int(cl(\tilde{B}))$,So, f is a fuzzy δ-continuous function.

THE COMPOSITION OF FUZZY CONTINUOUS FUNCTIONS

First, we study the composition of fuzzy $\theta - continuous$, fuzzy strong $\theta - continuous$, and

fuzzy δ-continuous.

Theorem 4.1. If $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$ and $g : (Y, \tilde{T}') \longrightarrow (Z, \tilde{T}'')$ are a fuzzy θ-continuous functions, then so is $gof : (X, \tilde{T}) \longrightarrow (Z, \tilde{T}'')$.

Proof:

Let $\tilde{C}$ be a fuzzy open set in $(Z, \tilde{T}'')$ since g is a fuzzy θ-continuous function.

Therefore $[g^{-1}(\tilde{C})]_\theta \subseteq g^{-1}(cl(\tilde{C}))$ (Theorem (3.3), part f). But $[g^{-1}(\tilde{C})]_\theta$ is the set of all a fuzzy

θ-cluster point of $g^{-1}(\tilde{C})$, there exist a fuzzy point (called fuzzy θ-cluster point) of a fuzzy set $g^{-1}(\tilde{C})$ s.t for every q-nbd. $\tilde{U}$ of $\tilde{p}$, $cl(\tilde{U})$ is q – coincident with $g^{-1}(\tilde{C})$ in $(Y, \tilde{T}')$.

Now, we have f is a fuzzy θ-continuous function, $[f^{-1}(g^{-1}(\tilde{C}))]_\theta \subseteq f^{-1}(g^{-1}(cl(\tilde{C})))$

$[(gof)^{-1}(\tilde{C})]_\theta \subseteq (gof)^{-1}(cl(\tilde{C}))$ (since $f^{-1}(g^{-1}(cl(\tilde{C})) = (gof)^{-1}(cl(\tilde{C}))$.

Hence (gof) is a fuzzy θ-continuous function (Theorem (3.3) (f)⇒(a)).

Theorem 4.2. If $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$ and $g : (Y, \tilde{T}') \longrightarrow (Z, \tilde{T}'')$ are a fuzzy strong θ-continuous functions, then so is $gof : (X, \tilde{T}) \longrightarrow (Z, \tilde{T}'')$.

Proof:

Let $\tilde{C}$ be a fuzzy open set in $(Z, \tilde{T}'')$ since g is a fuzzy strong θ-continuous function. Hence $g^{-1}(\tilde{C})$ is a fuzzy θ-open in $(Y, \tilde{T}')$ (Theorem (3.4), part (e)). Now f is a fuzzy strong θ-continuous function.

$f^{-1}(g^{-1}(\tilde{C}))$ is a fuzzy θ-open in $(X, \tilde{T})$.But, $f^{-1}(g^{-1}(\tilde{C})) = (gof)^{-1}(\tilde{C})$

This means that $(gof)^{-1}(\tilde{C})$ is a fuzzy θ-open in $(X, \tilde{T})$.Hence, (gof) is a fuzzy strong θ-continuous function.

Theorem 4.3. If $f : (X,\tilde{T}) \longrightarrow (Y,\tilde{T}')$ and $g : (Y,\tilde{T}') \longrightarrow (Z,\tilde{T}'')$ are fuzzy δ-continuous function, then so is $gof : (X,\tilde{T}) \longrightarrow (Z,\tilde{T}'')$.

Proof:

Let $\tilde{V}$ be a fuzzy regularly open set in $(Z,\tilde{T}'')$ since g is a fuzzy δ-continuous. Hence $g^{-1}(\tilde{V})$ is a union of a fuzzy regularly open sets in $(Y,\tilde{T}')$. Every a fuzzy regular open set is fuzzy δ-open set (Remark (2.4)).Therefore $g^{-1}(\tilde{V})$ is a fuzzy δ-open set.

Assume that $g^{-1}(\tilde{V}) = \bigcup_{\alpha\in\Omega} \tilde{W}_\alpha$, where $\tilde{W}_\alpha$ is fuzzy regular open in $(Y,\tilde{T}')$.

Now, $f^{-1}(g^{-1}(\tilde{V})) = f^{-1}\left[\bigcup_{\alpha\in\Omega} \tilde{W}_\alpha\right] = \bigcup_{\alpha\in\Omega} f^{-1}(\tilde{W}_\alpha)$ (since If $\tilde{A}_i \subset X$, for every $i \in I$, then $f\left(\bigcup_{i\in I} \tilde{A}_i\right) = \bigcup_{i\in I} f(\tilde{A}_i)$ .Now, f is fuzzy δ-continuous, so $f^{-1}(\tilde{W}_\alpha)$ is fuzzy δ-open in $(X,\tilde{T})$.

Hence, $\bigcup_{\alpha\in\Omega} f^{-1}(\tilde{W}_\alpha)$ is fuzzy δ-open in $(X, \tilde{T})$.So $f^{-1}(g^{-1}(\tilde{V}))$ is fuzzy δ-open in $(X,\tilde{T})$

and $f^{-1}(g^{-1}(\tilde{V})) = (gof)^{-1}(\tilde{V})$.This means that $(gof)^{-1}(\tilde{V})$ is fuzzy δ-open in $(X,\tilde{T})$,

Hence (gof) is a fuzzy δ-continuous function (Theorem (3.5) (f)⇒(a)).

THE RESTRICTION OF FUZZY CONTINUOUS FUNCTIONS

In the see from, study the restriction of fuzzy continuous functions of fuzzy $\theta-continuous$, fuzzy strong $\theta-continuous$, and fuzzy δ-continuous and we state and proof with the following theorem.

Theorem 5.1. If f : $(X,\tilde{T})\longrightarrow(Y,\tilde{T}')$ a fuzzy δ-continuous function and A is open set in X , then

f/A : $(A,\check{T}_A)\longrightarrow(Y,\tilde{T}')$ is a fuzzy δ-continuous function.

Proof:

Let $\tilde{V}$ be a fuzzy regular open set in $(Y,\tilde{T}')$

Since f is a fuzzy δ-continuous function, then $f^{-1}(\tilde{V})$ is a fuzzy δ-open set in $(Y,\tilde{T})$ (Theorem (3.5) (f))

Now, $(f/A)^{-1}(\tilde{V}) = f^{-1}(\tilde{V})\cap A$, but A is open set in X, then $f^{-1}(\tilde{V})\cap A$.

Hence, $(f/A)^{-1}(\tilde{V})$ is a fuzzy δ-open in A so (f/A) is a fuzzy δ-continuous function (Theorem (3.5) part (f) ⇒ (a)).

Theorem 5.2. If f : $(X,\tilde{T})\longrightarrow(Y,\tilde{T}')$ is a fuzzy θ-continuous function and A is open set in X then f/A : $(A,\check{T}_A)\longrightarrow(Y,\tilde{T}')$ is a fuzzy θ-continuous functions.

Proof:

Let $\tilde{B}$ be a fuzzy open set in $(Y,\tilde{T}')$,Since f is a fuzzy θ-continuous function

Then $[f^{-1}(\tilde{B})]_\theta \subseteq f^{-1}(cl(\tilde{B}))$ is implies (Theorem (3.3) (f))

Now: $[(f/A)^{-1}(\tilde{B})]_\theta = [A\cap f^{-1}(\tilde{B})]_\theta$

$\subseteq A\cap f^{-1}(cl(\tilde{B}))$

But, $A \cap f^{-1}(cl(\tilde{B})) = (f/A)^{-1}(cl(\tilde{B}))$. Therefore, $[(f/A)^{-1}(\tilde{B})]_\theta \subseteq (f/A)^{-1}(cl(\tilde{B}))$

Hence, f/A is a fuzzy θ-continuous function (Theorem (3.3)(f)⇒(a)).

Theorem 5.3. If $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$ is a fuzzy strong θ-continuous function and A is open set in X, then $f/A : (A, \check{T}_A) \longrightarrow (Y, \tilde{T}')$.

Proof:

Let $\tilde{B}$ be a fuzzy open set in $(Y, \tilde{T}')$,Since f is a fuzzy strong θ-continuous function

Then $f^{-1}(\tilde{B})$ is a fuzzy θ-open in $(X, \tilde{T})$ (Theorem (3.4) (e)).

Now $(f/A)^{-1}(\tilde{B}) = A \cap f^{-1}(\tilde{B})$ but A is open set in X , then $f^{-1}(\tilde{B}) \cap A$ is θ-open in A.

Hence, $(f/A)^{-1}(\tilde{B})$ is a fuzzy θ-open in A , so f/A is a fuzzy strong θ-continuous function (Theorem (3.4) part (e) ⇒ (a)).

RELATIONSHIP BETWEEN FUZZY CONTINUOUS FUNCTIONS.

We introduce and prove the following theorem where the relationship between fuzzy θ-continuous, fuzzy δ-continuous and fuzzy strongly θ-continuous.

Theorem 6.1. Every fuzzy strong θ-continuous function is a fuzzy δ-continuous function.

Proof:

Let $f : (X, \tilde{T}) \longrightarrow (Y, \tilde{T}')$ be a fuzzy strong θ-continuous and let $\tilde{p}$ a fuzzy point in $(X, \tilde{T})$.

Let $\tilde{V}$ be a fuzzy open set in $(Y, \tilde{T}')$ containing $f(\tilde{p})$, then there exists a fuzzy open set $\tilde{U}$ in $(X, \tilde{T})$ containing $\tilde{p}$, such that $f(cl(\tilde{U})) \subseteq \tilde{V}$, Now $f(int(cl(\tilde{U}))) \subseteq f(cl(\tilde{U})) \subseteq \tilde{V}$ but, $\tilde{V} \subseteq cl(\tilde{V})$,

$\tilde{V} = int(\tilde{V}) \subseteq int(cl(\tilde{V}))$ .So, $f(int(cl(\tilde{U}))) \subseteq int(cl(\tilde{V}))$

Which means that f is a fuzzy δ-continuous function.

Theorem 6.2. Every fuzzy strong θ-continuous function is a fuzzy θ-continuous function.

Proof:

Let $\tilde{p}$ be a fuzzy point in (X, $\tilde{T}$) and let a fuzzy open q-nbd. $\tilde{V}$ of f($\tilde{p}$).

We must prove, there exists a fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$ such that f(cl($\tilde{U}$)) $\subseteq$ cl($\tilde{V}$).Since f is a fuzzy strong θ-continuous function. Therefore there exists a fuzzy open q-nbd. $\tilde{U}$ of $\tilde{p}$.

$\rightarrow$ f(cl($\tilde{U}$)) $\subseteq$ $\tilde{V}$.Now: $\tilde{V}$ $\subseteq$ cl($\tilde{V}$).$\Rightarrow$f(cl($\tilde{U}$)) $\subseteq$ $\tilde{V}$ $\subseteq$ cl($\tilde{V}$) $\Rightarrow$ f(cl($\tilde{U}$)) $\subseteq$ cl($\tilde{V}$).

Hence, f is a fuzzy θ-continuous function.

Theorem 6.3.[6] Every fuzzy δ-continuous function is fuzzy θ-continuous function .

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