Theoretical calculations of the differential cross section for ^{3}He (d,p)α nuclear fusion reaction

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Abstract

In our present work, we focus on study the reaction between hydrogen isotopes, which are the most important fuels for controlled fusion research, namely, the D-^{3}He reaction. As a source of protons with 14MeV.The strong dependence of the basically hot plasma parameters such as differential cross section ,reactivity and the energy for emitted protons, upon the total cross section, make the problems for choosing the desirable formula for the differential cross section, the main goal for this work.

Key words: D-^{3}He,differential cross section, fusion plasma, reactivity, S-factor

حسابات نظرية للمقطع العرضي التفاضلي للتفاعل النووي الاندماجي ^{3}He(d,p)^{4}He

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الملخص

في عملنا الحالي ركزنا على دراسة التفاعل بين نظائر الهيدروجين والتي تعد الوقود الاكثر اهمية لابحاث الاندماج النووي المسيطر عليها المسمى ^{3}He(d,p)^{4}He. كمصدر للبروتونات طاقتها 14MeV.الاعتمادية العالية الدرجة لعوامل البلازما

الحارة كالتفاعلية, ومعدل التفاعل, وطاقة البروتونات المنبعثة على المقطع العرضي الكلي للتفاعل النووي الاندماجي تجعل خصوصية بحث أو اختيار علاقات تجريبية ملائمة قدر الإمكان لهذه المقاطع العرضية وبالتالي إمكانية تحويرها نظريا لأجل الوصول إلى توافق عال جدا مع نظيرتها العملية المنشورة حديثا, والذي يعد من أهداف البحث الحالي.

كلمات مفتاحية: المقطع العرضي التفاضلي , بلازما الاندماج, التفاعلية , S-factor.

Introduction

D-^{3}He reaction is highly desirable because both Helium-3 and Deuterium are stable and the reaction produces a 14 MeV proton instead of a neutron and the proton can be shielded by magnetic fields. This means it is possible to make a fusion reactor using D-^{3}He reaction where the fuels and the reactions produce no radioactivity and the reaction occurs in magnetized plasma and can be engineered to release its energy directly into electricity. This means a very compact and efficient fusion power plant, suitable for aircraft and spacecraft is possible using D-^{3}He. Because of the lack of radioactivity for direct power conversion to electricity, the practical use of D-^{3}He fusion reaction for power is the long-range goal of most fusion researchers. [1]

The basic goals in Deuterium Helium -3 fusion are to use either a tokomak or inertial electrostatic confinement to control the fusion of D and ^{3}He to produce an energetically efficient and minimally radioactive fusion reaction as a source of electricity .Another significant benefit of using Deuterium and Helium -3 fuels is that the reaction products, being charged particles, can be manipulated by electric and magnetic fields and can consequently use for direct energy conversion.

The D-^{3}He reaction generates no neutrons and, this reaction is expected to be used in advanced fuel fusion reactor. Therefore, the D-^{3}He reactor has no wall replacement cost. In addition, no tritium breeding system is needed for the D-^{3}He reactor, but ^{3}He gas is rare. Because the reaction rate of the D-^{3}He is less, D-^{3}He reactor requires highly efficient confinement properties and operation at high ion temperatures. Furthermore, the power densities of D-^{3}He is smaller than that of the D-T reactor; thus, D-^{3}He reactor requires a large plasma volume. In terms of cost, D-D is disadvantageous in comparison with D-^{3}He and D-T reactors. [2]

Theory

The determination of cross sections for charged-particle reactions is of particular importance for astrophysics. Reaction rates serve as input to various astrophysical models such as primordial nucleosynthesis or stellar evolution. [3]

The nuclear fusion cross sections of proton and deuteron induced reactions at low energies are of particular interest from the points of view of the stellar nucleosynthesis and the nuclear energy production. For the D-^{3}He reaction and in our works we focus on empirical formula used in calculating the total cross sections for this fusion reaction given as follows.[4].

$$\sigma(E) = \frac{1}{E}\frac{A_0}{1 + B_1E + B_2E^2 + B_3E^3} \qquad (1)$$

Where $\sigma(E)$ is the total reaction cross section at the incident centre of mass energy E in (mb) , and The value E is the deuteron bombarding energy MeV in the centre- of-mass (CM) reference frame.

Table 1: Coefficient used to represent the energy dependence of the total cross section ^{3}He (d,p)^{4}He fusion reaction ref.[4]

Coefficient	^{3}He(d,p)^{4}He fusion reaction
A_0	7.479×10^1
B_1	-3.566×10^{-1}
B_2	5.899×10^{-2}
B_3	3.2605×10^{-3}

In order to calculate the differential cross section for this fusion reaction we use the equation below.[4].

$$\frac{d\sigma}{d\Omega} = \sum_{i=0}^{N} a_l' p_{i(\cos\theta_p)} \qquad (2)$$

Where a_l^{l} are energy dependent coefficients

The energy dependence of the coefficients $a_l(E)$ is described with 5th order polynomials

$$a_{l(E)} = \sum_{k=0}^{5} \alpha_{lk} E^k \qquad (3)$$

Table 2: Results on best fit parameters of polynomial of 5th order used to represent the energy dependence of coefficients of Legendre polynomials describing the angular distribution in the energy range 0.6–8.0MeV.ref. [4].

a_l^{i}	α_{l0}	α_{l1}	α_{l2}	α_{l3}	α_{l4}	α_{l5}
a_0	1.144	-2.219×10^{-1}	-1.131×10^{-1}	5.334×10^{-2}	-7.371×10^{-3}	3.347×10^{-4}
a_1	1.355×10^{-1}	-5.091×10^{-2}	3.964×10^{-2}	-1.305×10^{-2}	1.749×10^{-3}	8.150×10^{-5}
a_2	-2.866×10^{-1}	3.186×10^{-1}	-2.004×10^{-2}	-1.745×10^{-2}	3.436×10^{-3}	-1.823×10^{-4}
a_3	1.929×10^{-1}	-3.258×10^{-1}	2.194×10^{-1}	-5.988×10^{-2}	6.879×10^{-3}	-2.793×10^{-4}
a_4	-9.471×10^{-2}	1.121×10^{-1}	-3.647×10^{-2}	8.610×10^{-3}	-1.074×10^{-3}	5.007×10^{-5}
a_5	4.324×10^{-2}	-4.425×10^{-2}	2.179×10^{-2}	9.399×10^{-4}	-7.215×10^{-4}	4.933×10^{-5}
a_6	-6.944×10^{-2}	6.944×10^{-2}	-2.301×10^{-2}	4.434×10^{-3}	-2.541×10^{-4}	-6.346×10^{-7}

Fusion energy is expected to be a potential source for power production in the mid- to late-21st Century. The fusion reaction or process occurs within plasma composed of light nuclei. A commercial fusion power facility would likely use either magnetic or inertial means to confine the plasma and facilitate the fusion process. The fusion confinement method influences the radiation types and fuel materials that must be controlled by the health physicist. [5].

By the laws of conservation of energy and momentum, and from the general basic fundamental equation of calculating the energy for the emitted charge particles from any nuclear reaction given below [6].

$$\sqrt[2]{E_3} = v \pm \sqrt[2]{v^2 + \omega} \quad (4)$$

Where $v = \frac{\sqrt[2]{M_1 M_3 E_1}}{M_3+M_4} \cos\theta \qquad (5)$

$$and \qquad \omega = \frac{M_4\ Q + E_1\ (M_4 - M_1)}{M_3 + M_4} \qquad (6)$$

For the case of the binary $D - {}^{3}He$ fusion reaction,

$$\mathrm{D} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He}(3.712\ \mathrm{MeV}) + \mathrm{P}\ (14.641\ \mathrm{MeV})$$

Where

$\mathrm{E_3 = E_p}\ ,\mathrm{M_1 = M_d}\ ,\mathrm{M_3 = M_p}\ ,\mathrm{M_4 = M_{{}^4He}}\ ,\mathrm{E_1 = E_d}$

And $Q = 18.353\ MeV$ is the Q-value of the above D-He fusion reaction, E is the deuteron bombarding energy.

Substituting the values for the quantities $\nu, \omega, M_1, M_2, M_3, M_4, E_1, Q$ as it is described above in equation (4), and taken into account some mathematical analysis steps to get or deduced a formula for evaluating the energy of the emitted neutrons that is given below:

$$E_p = \frac{20\,Q + 12\,E_d}{25}\left[\sqrt[2]{1 - \gamma^2 \sin\theta} + \gamma\cos\theta\right]^2 \qquad (7)$$

Where $\gamma^2 = \frac{2\,E_d}{20\ Q + 12\ E_d}$

Equation (7), explain the relationship between the energy of the emitted proton from the $D - {}^{3}He$ fusion reaction with the energy of bombarding deuterons and reaction angle $E_d\ , \theta,$ respectively, and it's dominate recommended value is equal to 14.641 Mev.

The plasma astrophysical S-factor for the ^{3}He (D, p)^{4}He fusion reaction represent an important parameter because it characterizes the behavior of interactions within the plasma. Given in the form: [7].

$$S(E) = 18.52(1 - 4.697E + 39.53E^2 - 109.6E^3 + 130.7E^4 - 54.83E^5) \Big/ \left(1 + \left(\frac{E - E_R}{\Gamma_R/_2}\right)^2\right)$$

(8)

Where E_R =0.183 MeV and $\Gamma_R = 0.256$ MeV.E$_R$ and $\Gamma_R$are the resonance parameters.

Finally, by definition the reaction rate R for reaction i in a space independent problem is given by

$$R = n_D n_{He} \iiint f(\vec{v}_D) F(\vec{v}_{He}) g \frac{d\sigma_i}{d\Omega} d\Omega d\vec{v}_D d\vec{v}_{He} \qquad (9)$$

Where $n_D and n_{He}$ are the beam and target ions densities with unit-normalized distribution functions $f and F$ respectively; g is the modulus of the relative velocity between beam and target ions and $\frac{d\sigma_i}{d\Omega}$ is the doubly differential cross section.

In the Bernstein – Comisar formalism one can introduce the dummy variable $E$by use of a Dirac Delta distribution, so that

$$R - n_D n_{He} \iiint f(\vec{v}_D) F(\vec{v}_{He}) g \frac{d\sigma_i}{d\Omega} \delta\left(E - E_p\right) d\Omega d\vec{v}_D d\vec{v}_{He} dE \qquad (10)$$

The plasma reactivity for the ^{3}He(D, p)^{4}He fusion reaction as a function of the temperature, obtained by numerical integration of the following equation with the best available cross section for the reactions of interest to controlled fusion.

$$\langle \sigma v \rangle = \left[\left(\frac{m_1 + m_2}{2\pi k_B T} \right)^{3/2} \int dV_c \exp\left(- \frac{m_1 + m_2}{2k_B T} V_c^2 \right) \right] \times \left(\left(\frac{m_r}{2\pi k_B T} \right)^{3/2} \int dV_c \exp\left(- \frac{m_r}{2k_B T} V^2 \right) \sigma(v) v \right)$$

The term in square bracket is unity, being the integral of a normalized Max wellain, and we are left the integral over the relative velocity. By writing the volume element in velocity space as $= 4\pi v^2 dv$, and using the definition of center of mass energy ϵ , we finally get

$$\langle \sigma v \rangle = \frac{4\pi}{(2\pi m_r)^{1/2}} \frac{1}{(k_B T)^{3/2}} \int_0^{\infty} \sigma(\in) \in \exp(-\in / k_B T) d$$

For the $D - {}^{3}_{2}He$ fusion reaction, which is by far the most important one for present fusion research, the following expression is used to calculate the reactivity [8]. Where T is the temperature in (keV).

$$\langle\sigma v\rangle_{D-3He} = 4.98\times 10^{-16}\exp\left(-0.152\left|\ln\frac{T}{802.6}\right|^{2.68}\right) \qquad (11)$$

There is another formula to calculate the reactivity for D-^{3}He fusion reaction. [9].

where ξ is Somerfield parameterr.

$\langle\sigma v\rangle_{D-3He} = C1\,.\,\vartheta\sqrt{\zeta/(m_r c^2 T^3)}e^{-3\xi}$ (12)

Where $\theta = T/\left[1-\frac{T(C2+T(C4-TC6))}{1+T(C3+T(C5+TC7))}\right]$(13)

$\xi = \left(B_G^2/(4\theta)\right)^{1/3}$(14)

Table 3: Coefficients used in the equations for the determination of the reactivity. ref. [9]

coefficient	^{3}He(d,p)^{4}He
$B_G\sqrt{(KeV)}$	68.7508
$m_r c^2$(KeV)	1124572
C1	5.51036E-10
C2	6.41918E-3
C3	-2.02896E-3
C4	-1.91080E-5
C5	1.35776E-4
C6	0.0
C7	0.0

Calculation and results

The calculations are concentrated on the D-^{3}He fusion reaction, because of its huge applications in power source due to their high energy release, such as fusion reactors, (Tokomak), and others small systems like the dense plasma focus devices (DPF).

As it is described previously, in order to calculate such parameters, i.e., differential cross sections, reactivity, are all controlled by cross section, which represents the essential factor in the calculations. By testing equation (1), for the total cross section for D-^{3}He fusion reaction we arrive to results gives high agreement with the corresponding experimental published results in ref. [4].As shown in figure (1).

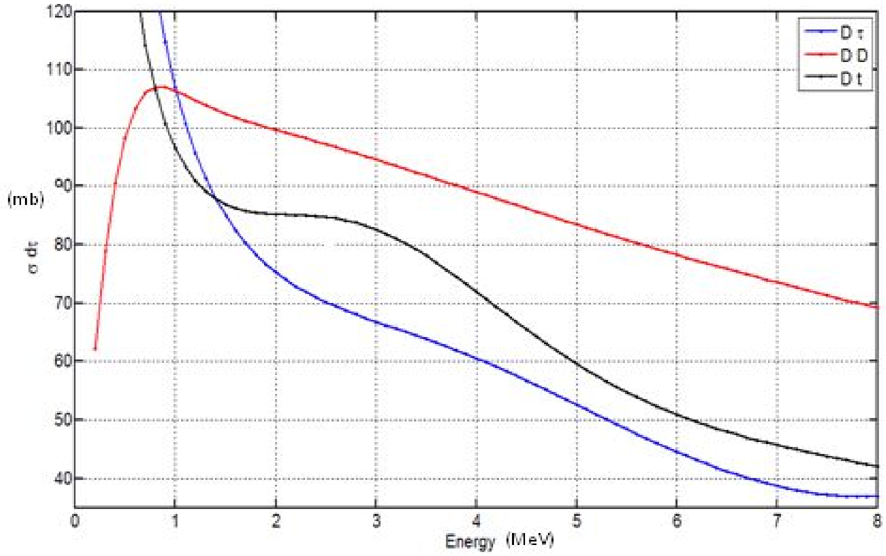

Fig (1): The total cross sections *σ (E)* for the fusion reactions Dτ, DD, Dt as a function of deuteron energy.

The differential cross sections dσ/dΩ are calculated as a function of reaction angle θ for deuteron energies

Range (1-8)MeV by using equation (2).The results of calculations dσ/dΩ and θ is presented in figures (2) and (3).

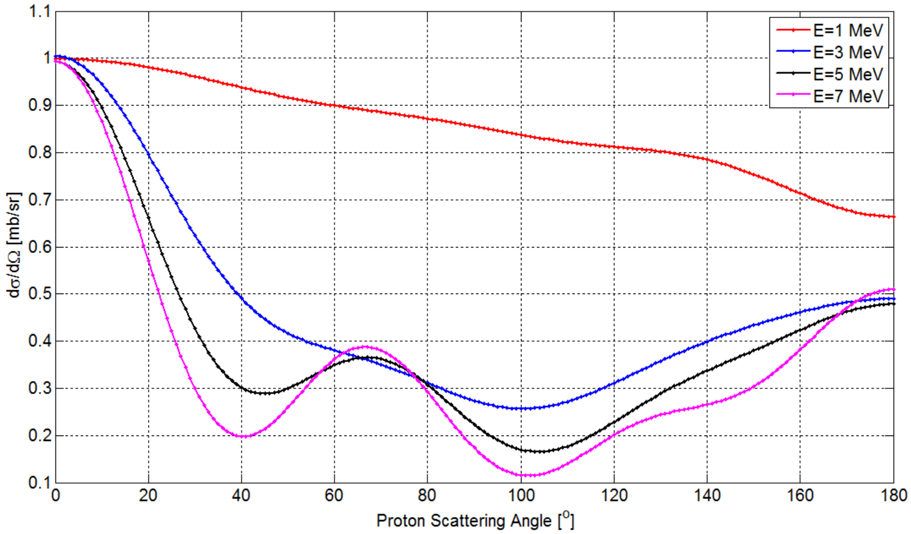

Fig (2): Show the calculated differential cross-sections for the $^{3}He(d,p)^{4}He$ reaction as a function of proton scattering angle.

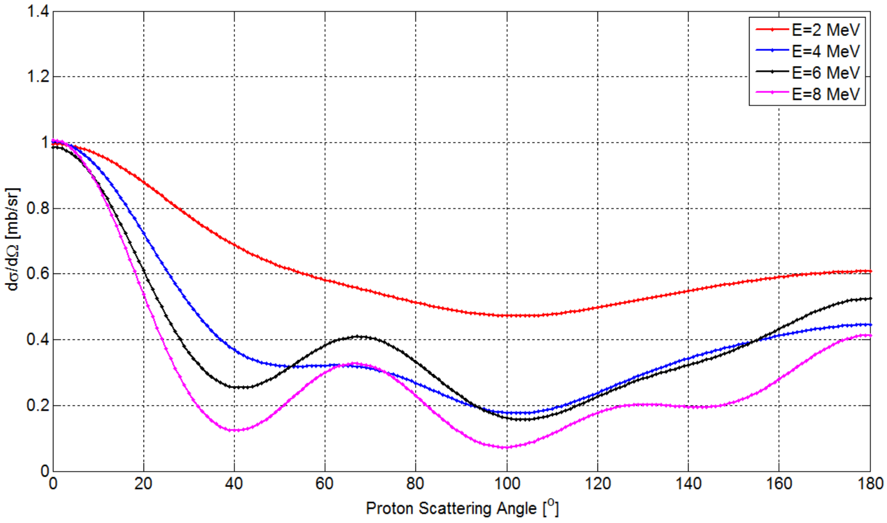

Fig (3): Show the calculated differential cross-sections for the $^{3}He(d,p)^{4}He$ reaction as a function of proton scattering angle.

From the comparison of reactivities between present work and the results published in ref.(9) we found a good agreement between them as shown in fig(4) and Table(4).

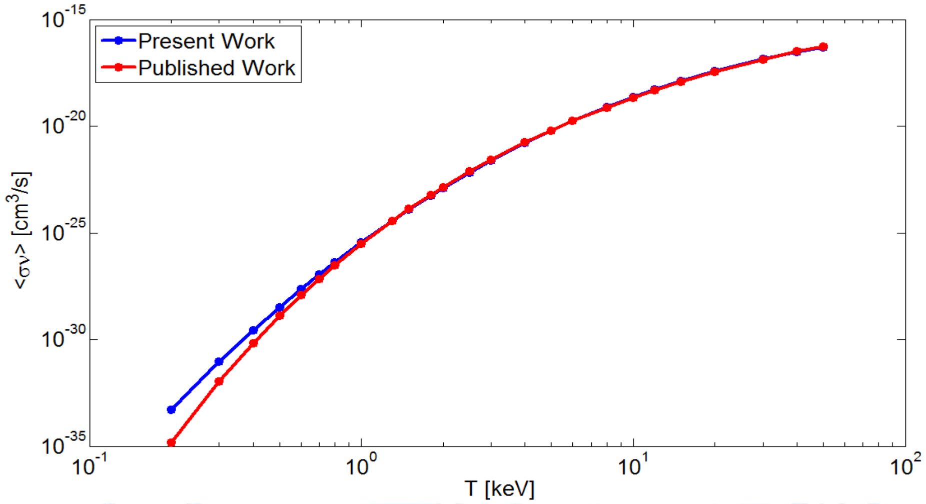

Figure (4): Variation of the D-^{3}He Reactivity versus the incident deuteron temperature between present work and published work ref. [9]

Table 4: Thermal Reactivates for ^{3}He(d,p)^{4}He reaction as a function of the ion temperature.

T(KeV)	^{3}He(d,p)^{4}He (cm^3/s) present work	^{3}He(d,p)^{4}He (cm^3/s) published work ref.[9]
0.2	5.204e-34	1.414E-35
0.3	8.953e-32	1.033E-32
0.4	2.676e-30	6.537E-31
0.5	3.240e-29	1.241E-29
0.6	2.271e-28	1.166E-28
0.7	1.107e-27	6.960E-28
0.8	4.173e-27	3.032E-27
1	3.493e-26	3.057E-26
1.3	3.672E-25	3.708E-25
1.5	1.241E-24	1.317E-24
1.8	5.494E-24	6.053E-24

2	1.255E-23	1.399E-23
2.5	6.693E-23	7.477E-23
3	2.432E-22	2.676E-22
4	1.624E-21	1.710E-21
5	6.332E-21	6.337E-21
6	1.793E-20	1.739E-20
8	8.174E-20	7.504E-20
10	2.392E-19	2.126E-19
12	5.396E-19	4.715E-19
15	1.352E-18	1.175E-18
20	3.925E-18	3.482E-18
30	1.418E-17	1.363E-17
40	3.054E-17	3.160E-17
50	5.123E-17	5.554E-17

From the calculation of S-factor we see a good agreement with published in ref. (7) as shown in figure (5) and Table(5).

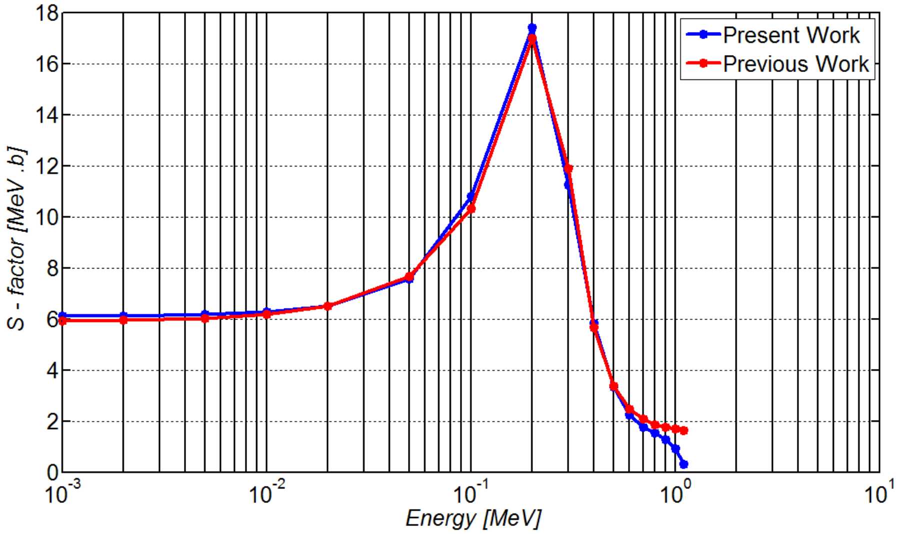

Fig (5): Comparison between S-factor as a function of deuteron energy between present work and published results ref. [7].

Table 5: Results of S-factor as a function of deuteron energy.

E(MeV)	S-factor (MeV b) present work	S-factor(MeV b) Published result ref.7
0.001	6.1	5.92
0.002	6.11	5.95
0.005	6.17	6.03
0.01	6.26	6.19
0.02	6.5	6.51
0.05	7.57	7.67
0.1	10.8	10.3
0.2	17.4	17
0.3	11.24	11.9
0.4	5.81	5.65
0.5	3.34	3.36
0.6	2.25	2.47
0.7	1.77	2.07
0.8	1.524	1.87
0.9	1.298	1.76
1	0.933	1.68

Theoretically, the energy of the emitted protons from the $D-{}^{3}He$ fusion reaction can be exactly determined from equation (7) as a function of both the incident deuteron energy $E_d$and the reaction angleθ.the calculated results are completely described in Table (6) and completely described in figure (6).

Table 6: The calculated energy of emitted neutrons as a function of the reaction angle.

E_d(keV) $\theta = 0^0$	E_p(MeV)	E_d(keV) $=60^0\theta$	E_p(MeV)	E_d(keV) $\theta=90^0$	E_p(MeV)
20	15.000	20	14.845	20	14.691
30	15.075	30	14.884	30	14.695
40	15.138	40	14.917	40	14.699
50	15.195	50	14.947	50	14.703
60	15.247	60	14.975	60	14.707
70	15.296	70	15.000	70	14.711
80	15.341	80	15.025	80	14.715
90	15.384	90	15.048	90	14.719
100	15.425	100	15.069	100	14.723

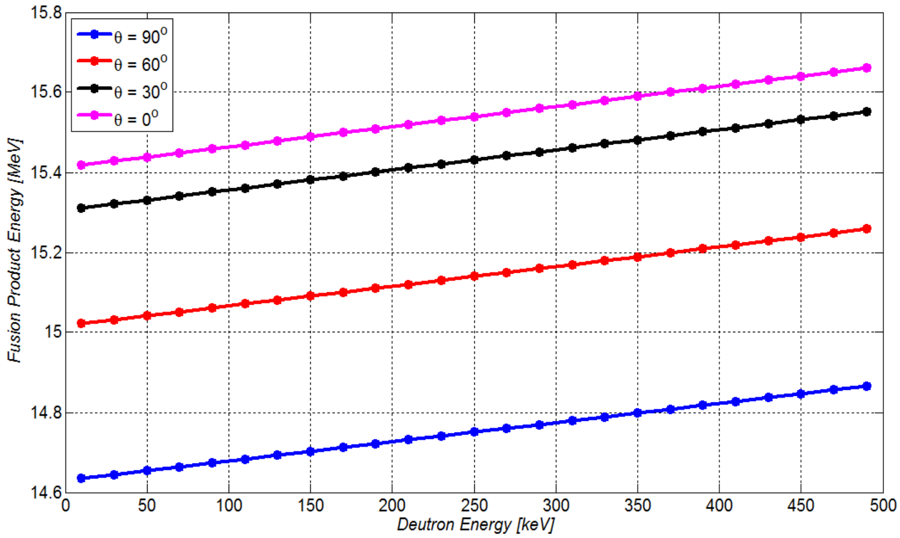

Figure 6: Variation of the emitted protons energy versus the incident deuteron energy.

Discussion and Conclusion

From the results about the total cross section for the $D - {}^{3}_{2}He$ fusion reaction calculated by equation (1), it is appear a common shift from the published experimental results and one can interpreted that by some physical reasons that directly correlated with the fundamentals parameters deal with the system or device designing, geometrical dimension for the cathode and anode, and the operating factors, such as the fuel pressure, initial power, and we can added another reason deal with the construction time for building the experiment, in which that any system are exactly differ in all covering physical conditions with the recent ones. In other words, it is necessary to give empirical formula for each system (experimental devices). By comparing the differential cross section for this reaction at deuteron energy as presented in Fig.(2,3) with the corresponding experimental published ref.(4) .We see a good agreement between our results and experimental published and this behaviour leads to a fact that there exists a real precession for the choices of differential cross section formula which in turn gives reflect on the results about both the two other parameters, respectively. And the above conclusions are very clearly shown in the physical behaves for the D-^{3}He reaction rate in which we have very similar shapes with the other published results in ref.9.

From the comparison of S-factor between the results of present work and published results in ref.7 we find a good agreement between them as shown in figure (5) and Table(5). From Table (6), it is clear that the energies of emitted proton, which calculated by our expressed formula at incident reaction angle of 90^{o} are of good agreement with the recommended value of ($14.641\ MeV$), and this case can be interpreted as, there exist a small percentage of incident deuterons are scattered from its original direction, and all the really occurring physical experimental phenomenon's can be explained at this angle instead of others angles. It is necessary to denoted that there are certain conditions for any experiment and any given empirical formula are concentrated by the rules of varying the basic parameters exist or effect the equation, until they reach to a formula that agreed with the measured data.

Finally, the attractiveness of D-^{3}He fusion's engineering, safety, and environmental characteristic makes this potentially important research area.

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