# Calculations of Ground Band in even-even $^{170-180}W$ nuclei by Interacting boson model (IBM-1)

**Mushtaq Abed Dawood Al-Jubbori* Hussein Ali Hassan Al-Saffar****
* Department of Physics, College of Education for Pure Science, Mosul University, Iraq
**Ministry of Education , General Directorate for Nineva Education, Iraq

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## Abstract

In this work, the properties of $^{170-180}W$ isotopes are studied by the E-GOS curves and the relation between energy levels and $E_2$ showed that $^{170-180}W$ lie in SU(3)-O(6) transition region were investigated to calculate the energy levels of ground state band according to the Interacting Boson Model (IBM-1). The entire calculations and drawing of the figures implemented by one program written by matlab language. The results are compared with experimental data and showed good agreement.

**Keywords:** energy levels, IBM(1), SU(3)-O(6), E-GOS, W isotopes.

## حساب مستويات الطاقة للحزمة الأرضية لنظائر $^{170-180}W$ الزوجية – الزوجية بأستخدام أنموذج البوزونات المتفاعلة (IBM-1)

**مشتاق عبد داؤد الجبوري* حسين علي حسن الصفار****
*قسم الفيزياء، كلية التربية للعلوم الصرفة، جامعة الموصل، العراق
**وزارة التربية، المديرية العامة لتربية نينوى، العراق

# Calculations of Ground Band in even-even $^{170-180}W$ nuclei by Interacting boson model (IBM-1)

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## الخلاصة

في هذا البحث تم دراسة خصائص نظائر التنكستن $^{170-180}W$ بواسطة منحنيات كاما مقسوماً على الزخم E-GOS وتبين من العلاقة بين مستويات الطاقة ومستوي $E_2$ ان هذه النظائر تقع بين التحديدين SU(3)-O(6) أي في المنطقة الأنتقالية بين النوى الدورانية والنوى ذات خصائص كاما الناعمة. حسبت مستويات الطاقة للنظائر المدروسة بأستخدام انموذج البوزونات المتفاعلة IBM-1 من خلال برنامج حاسوبي كتب بلغة matlab يفي بهذا الغرض من اجراء الحسابات ورسم الأشكال المطلوبة. قورنت الحسابات الحالية مع النتائج العملية وتبين انها تتفق بشكل جيد.

**الكلمات المفتاحية:** نظائر التنكستن, منحنيات E-GOS، SU(3)-O(6) , أنموذج البوزونات المتفاعلة IBM-1، مستويات الطاقة.

## Introduction

Several phenomenological and geometrical models [1] have been proposed to investigate the nuclear structure by the prediction of the ground states and the description of electromagnetic transition rates [2]. The quadrupole correlations and electromagnetic transition in nuclei depends mainly on the neutron-proton interaction. However, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell [3,4]. The tungsten isotopes received considerable attention both theoretically and experimentally in recent years. Abdul Ameer and Al-Shimmary [5] calculated the energy levels, B(E2) transition probabilities and electric quadrupole moment of the even-even $^{180-190}W$ isotopes in the transition region SU(3)-O(6). The energy levels, electric quadrupole moments, B(E2) values of $^{182-186}W$ isotopes have been calculated by, Salem *et al* [6] within the framework of the interacting boson model IBM-2. Due to the increased interest in this subject recent years, the energy levels and E-Gos of even-even $^{170-180}W$ isotopes in SU(3)-O(6) transition region within framework are studied by the Interacting Boson Model (IBM-1) .

## Theory

**Energy Gamma Over Spin (E-GOS)**

Many nuclei exhibit a decay sequence consistent with quasi-vibrational excitations at lower spins, the perfect harmonic vibrator of gamma-ray decay energies are given by :

$$E_\gamma(J \rightarrow J-2) = \hbar w \quad (1)$$

where $J$ is spin state, $\hbar$ is plank's constant and $w$ is angular frequency. While, for an axially symmetric rotor,

$$E_\gamma(J \rightarrow J-2) = \frac{\hbar^2}{2\vartheta}(4J-2) \quad (2)$$

where $\vartheta$ is moment of inertia of the nucleus.

The gamma-soft nucleus can be written as

$$E_\gamma(J \rightarrow J-2) = \frac{E2_1^+}{4}(J+2) \quad (3)$$

The ratio $R = \frac{E_\gamma(J \rightarrow J-2)}{J}$ provides an effective way of distinguishing axially symmetric rotational, $\gamma$-unstable and harmonic vibrational mode [7].

at $J \rightarrow 0$

for vibrator $R = \frac{\hbar w}{J} \rightarrow 0 \quad (4)$

Rotor $R = \frac{\hbar^2}{2\vartheta}(4 - \frac{2}{J}) \rightarrow 4\frac{\hbar^2}{2\vartheta} \quad (5)$

$\gamma$-unstable $R = \frac{E2_1^+}{4}(1 + \frac{2}{J}) \rightarrow \frac{E2_1^+}{4} \quad (6)$

Figure (1) shows these theoretical limits plotted for three schematic nuclei: (i) a vibrator in which the first $2^+$ excited state lies at an energy of 500 keV, (ii) a rotor where this energy is 100 keV and (iii) a $\gamma$-unstable of energy 300 keV (These values were taken to represent typical nuclear vibrator, rotor and $\gamma$-unstable energies, respectively.)

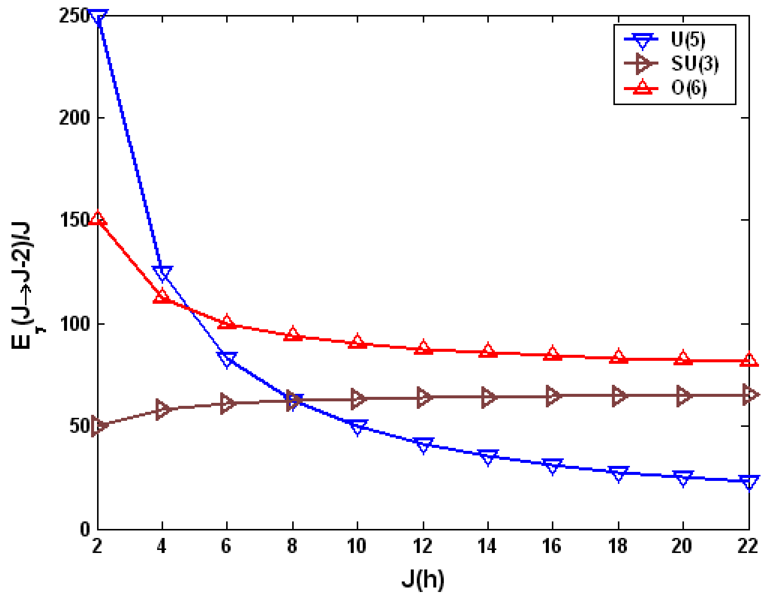


**Figure (1): Standard curve of E-GOS plot for vibrational U(5), gamma unstable O(6) and rotational bands SU(3)**

## Interacting Boson Model (IBM-1)

In the present work, the IBM-1 states of the low lying collective state of even-even nuclei can be described by the interaction of s and d−bosons, carrying angular momentum L = 0 and L= 2, respectively. The IBM-1 Hamiltonian is written

$$H = \varepsilon'' \hat{n}_d + a_0 \hat{P}^+ \hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \quad (7)$$

Where $a_o$ , $a_1$ , $a_2$ , $a_3$ and $a_4$ are strength of pairing, angular momentum and multipole terms. The Hamiltonian tends to reduce into three limits, the vibration U(5), γ-soft O(6) and the rotational SU(3) nuclei [8]. In U(5) limit, the effective parameter is ε, in the γ-soft limit O(6) the effective parameter is the pairing $a_o$ and in the SU(3) limit the effective parameter is the quadrupole $a_2$ .

The eigenvalues for the SU(3)-O(6) limit is given by [9]

$$E(\lambda,\mu,J) = K_3[N(N+4) - \sigma(\sigma+4)] + K_2(\lambda^2 + \mu^2 + 3(\lambda+\mu) + \lambda\mu) + K_5 J(J+1) \quad (8)$$

for low-laing N=σ, μ=0 and λ=2N therefore

$$E(\lambda, J) = K_2(\lambda^2 + 3\lambda) + K_5 J(J+1) \quad (9)$$

## Discussion and Conclusions

The parameters of Eq.(9) were calculated for $^{170-180}W$ nucleus and listed in table (1), it was found that $K_2$ decreases and $K_5$ increases with mass number.

**Table(1): The Parameters $K_2$, $K_5$ and the ratio $R=E_4/E_2$ for $^{170-180}W$ isotopes**

| Nucleus | $K_2$ (keV) | $K_5$ (keV) | $R=E_4/E_2$ |
|---|---|---|---|
| $^{170}W$ | 25.7672 | 4.1226 | 2.95 |
| $^{172}W$ | 18.4796 | 5.2798 | 3.0609 |
| $^{174}W$ | 13.9465 | 7.7379 | 3.154 |
| $^{176}W$ | 12.6059 | 8.4789 | 3.2151 |
| $^{178}W$ | 10.7882 | 9.5505 | 3.2364 |
| $^{180}W$ | 10.4675 | 9.6598 | 3.26 |

Figure (2) shows that the ratio $E_J/E_2$ of $^{170-180}W$ versus spin (J) in which the limit lies between SU(3)-O(6). Eq.(9) used to calculate the energy levels of ground state band.

Figure (3) shows the E-GOS curves of the ground states band of isotopes. Comparing these curves with the ideal limits of vibration, rotational and γ -soft show the evolution in the property of these isotopes, were the slow reduce of all the curves from the first to the last excited states confirms that the $^{170-180}W$ isotopes lies between SU(3)-O(6) limit.

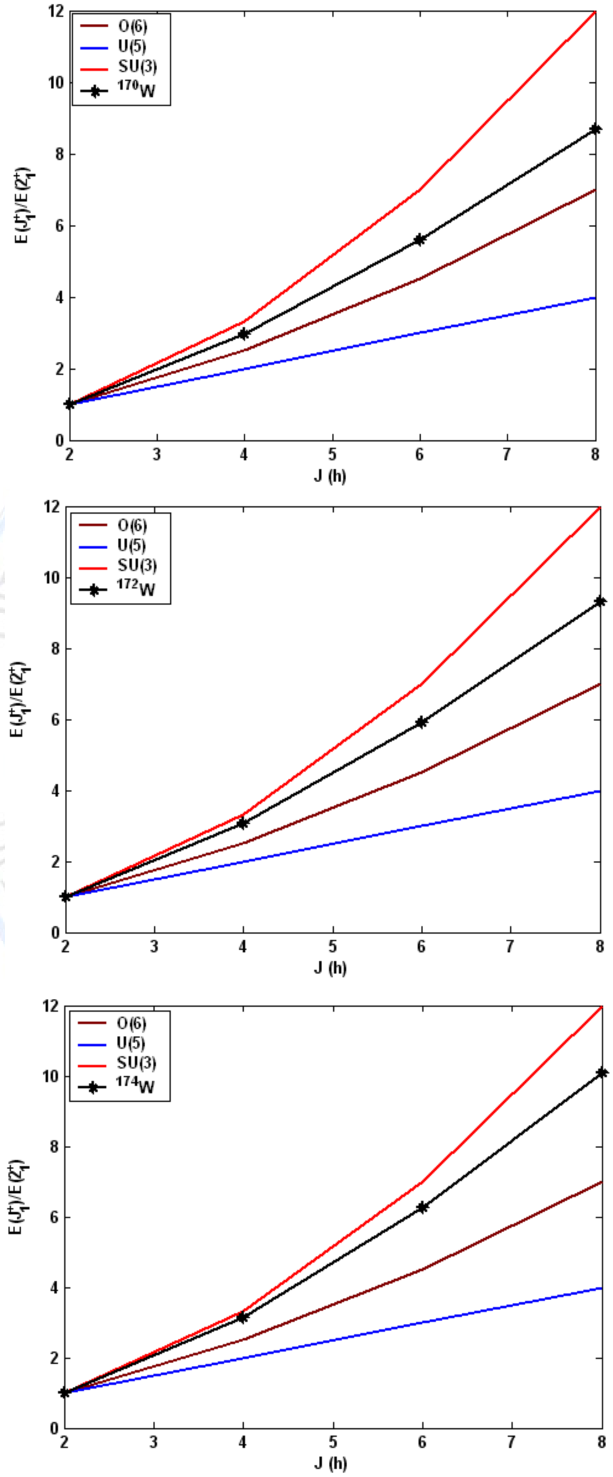


**Figure (2): E(J$_1$)/E(2$_1$) versus spin J**

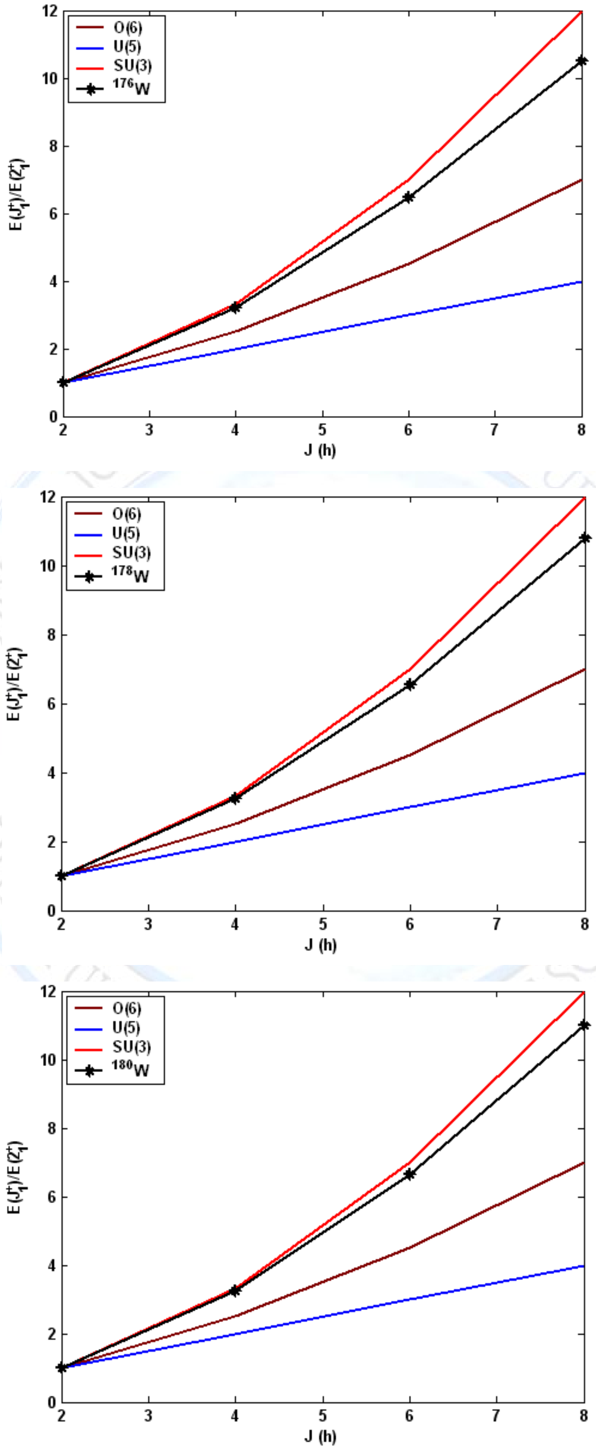


**Fig. 2. (continued)**

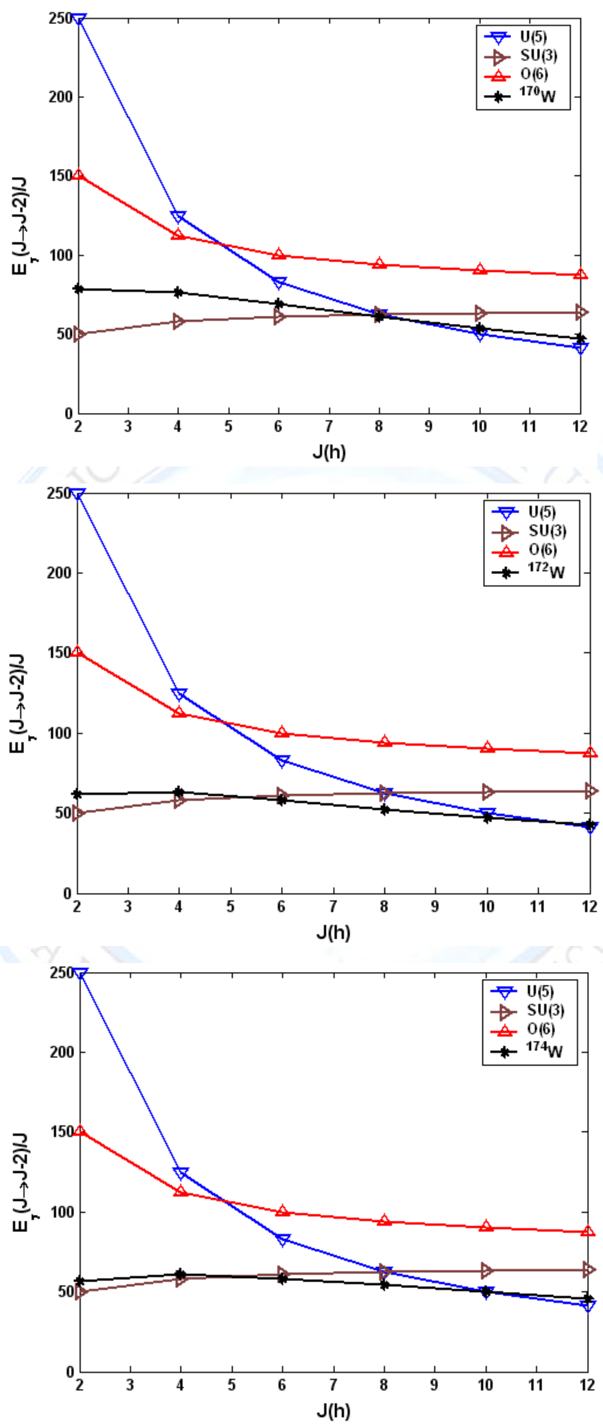


Figure (3): The E-GOS of $^{170-180}W$ isotopes.

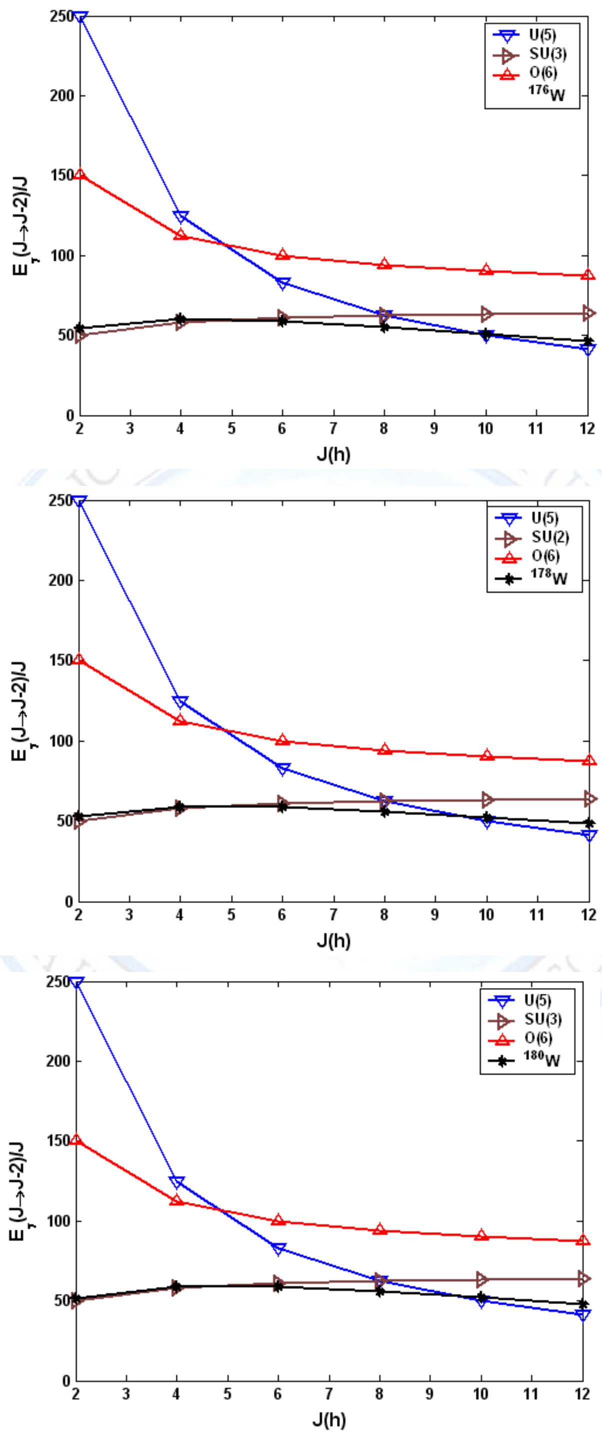


**Fig. 3. (continued)**

Figure (4) shows the present calculations of the energy levels were found to be in a reasonable agreement with measured values.

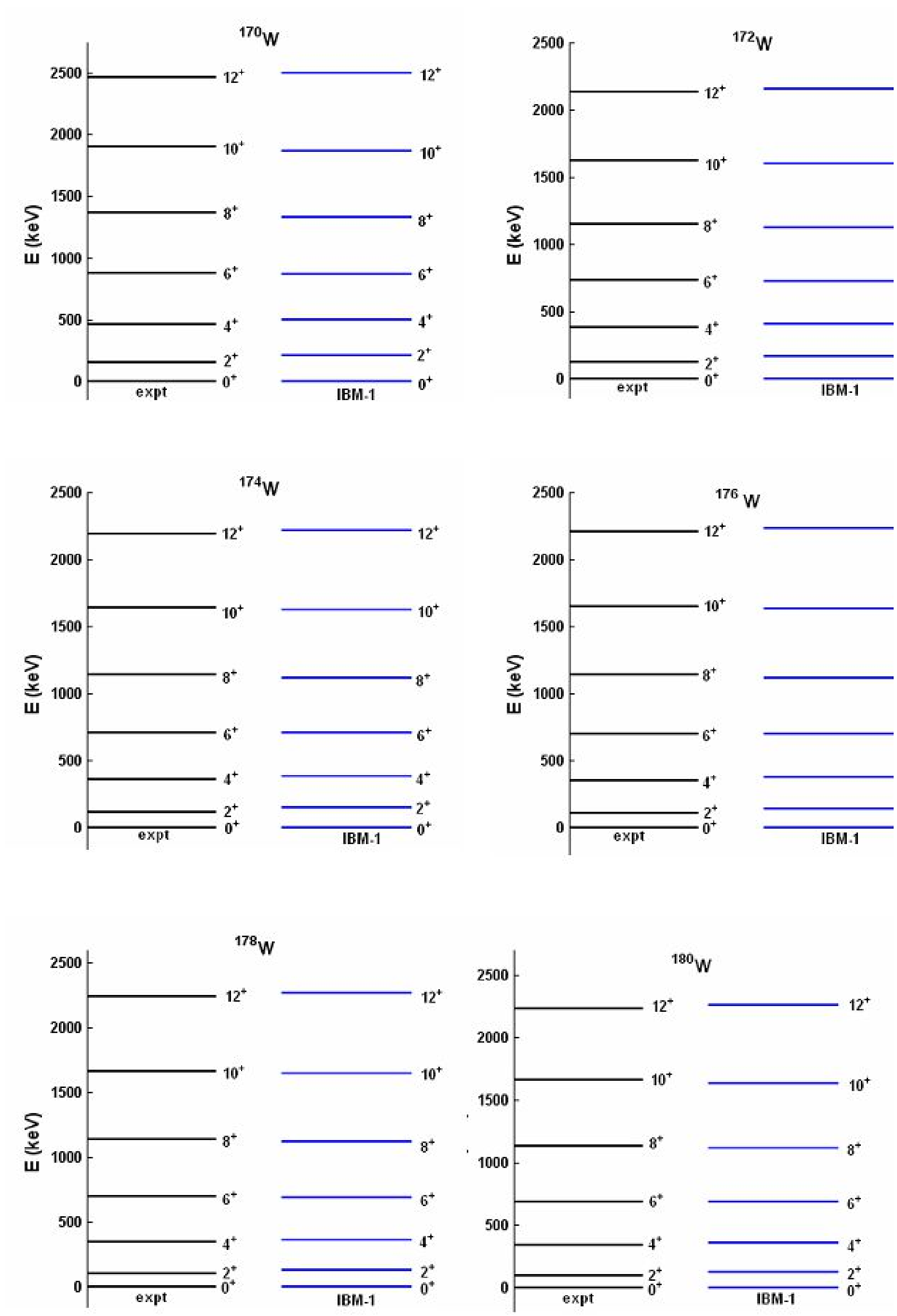


**Figure (4): Comparison between experimental energy levels and calculated data**

## References

1. H. M. Mittal and Vidya Devi, Armen. J. Phys. vol. 2, issue 3, pp.146-156, (2009).
2. Hossain, Hewa Y. Abdullah, I. M. Ahmed, M. A. Seed, S. T. Ahmad. Armen. J. Phys. vol. 5, issue 3, pp. 101-104, (2012).
3. R. F. Casten, Nucl. Phys. A. vol. 347,pp. 173-204, (1980).
4. M. A. D. Al-Jubbori, Dialya Journal Pure. Science. vol. 9 No: 2, pp.45-56, (2013).
5. M. Abdul Ameer and M. A. H. Al-Shimmary. Armenian Journal of Physics, vol. 4, issue 3, pp. 146-153, (2011).
6. A. M. Salem, S. Olaf and S. M. Daw. the Fourth International Conference of Science & Development, Gaza (2011) (Abstract).
7. P. H. Regan, C.W. Beausang, N.V. Zamfir, R. F. Casten, Jing-yeZhang, A. D. Yamamoto, M. A. Caprio, G. Gurdal, A. A. Hecht, C. Hutter, R. Krucken, S. D. Langdown, D. A. Meyer, and J. J. Ressler. Phys. Rev. Lett. 90, pp. 152502, (2003).
8. A. Arima and F. Iachello, Phys. Rev. Let., 40, pp.385-391, (1978).
9. NSDD Workshop, Triste, (2005).