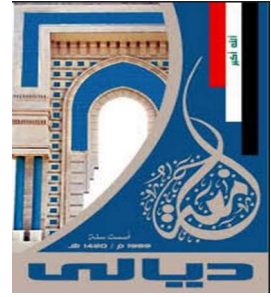


**Ministry of Higher Education  
and Scientific Research  
University of Diyala  
College of Engineering**



# **EXPERIMENTAL AND NUMERICAL STUDY OF ELASTOHYDRODYNAMIC LUBRICATION**

**A Thesis Submitted to the Council of College of Engineering,  
University of Diyala in Partial Fulfillment of the Requirements  
for the Degree of Master of Science in Mechanical Engineering**

**by**

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We certify that we have read the thesis entitled “**Experimental and Numerical Study of Elastohydrodynamic Lubrication**” and we have examined the student (**Hassan Sami Fatehallah**) in its content and what is related with it ,and in our opinion it is adequate as a thesis for **Degree of Master of Science in Mechanical Engineering.**

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# **DEDICATION**

**This study is dedicated to:**

**The teacher of human knowledge of  
our Prophet Muhammad (peace be upon him)**

**My father and my mother**

**My family and friends, with love, respect and  
gratitude.**

## **ACKNOWLEDGEMENT**

**Firstly, I** want to thank (ALLAH) for giving me the impatient and incurring to finish this study, I would like to thank the people and sponsors who made this study possible to finish it.

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I am truly grateful for all that, they have done.

## ABSTRACT

Experimental and Numerical Study of Elastohydrodynamic Lubrication

by

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Supervised by

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Prof. Dr. Lutfi Y. Zedan

An introduction of “Elastohydrodynamic lubrication” EHL has presented in this study. A specification of the main mathematical model has also presented. Two kinds of EHL problems are considered, line and point contacts. In this study, numerical techniques are developed to solve EHL for lightly, moderately, and highly loaded cases. The discretization of equations was done using two approaches, “Finite Element Method” FEM, and “Finite Difference Method” FDM. For lightly loaded case, the direct coupling technique was used. The moderately loaded case was solved using “Forward Iterative Technique”, with central and gauss-Siedel iteration. For this technique, the load can be reached approximately for line contact 650 MPa, and for point contact 850 MPa. For highly loaded case, “Newton-Raphson” technique was used to solve line contact EHL problem. The load can be reached approximately 2000 MPa. The developed program of point contact was used with a range of temperature (-20 to 120 °C) to study the effect of temperature on EHL. The increasing of temperature causes a decrease in film thickness and friction coefficient. The minimum film thickness decreases gradually approximately by 90.2%, because of the decreases of viscosity. Where the minimum film thickness is directly proportional with the viscosity as in Hamrock equation. The friction coefficient decreases approximately by 96.4%, because of the decreases of lubricant viscosity. For experimental analyses, a ball on disc test rig is built to investigate the friction coefficient of point contact EHL problem. A number of engine oils were used as

lubricants to make tests of measure friction coefficient. The effects of number of parameters, such as oil temperature, oil type, and loads on the pressure, film-thickness and friction coefficient are analyzed. At the same load and same initial conditions the results shows, the increase in friction coefficient with the increase of viscosity is almost linear with an average slope of  $1.4 \text{ (Pa.s)}^{-1}$ . Also, the increase in load results in linear increase in friction coefficient by an average slope of  $0.63 \text{ N}^{-1}$ . The experimental results are compared with the numerical results, and the average of the deviation between the results are 58%.

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## LIST OF ABBREVIATIONS

Symbol	Definition
a	the half-width in the point contact
b	the half-width in the line contact
$D_{ij}^{kl}$	The two-dimensional elastic deformation stiffness.
$e_{l_{top}}$	The element topology
E	The equivalent elastic modulus of the two contact surface materials.
EHL	Elastohydrodynamic lubrication
F	The Friction force (N)
FEM	Finite element method
FDM	Finite difference method
geom	The geometry of nodes (x,y)
$G^*$	A material parameter, $G = \alpha E$ .
h	The oil film-thickness (m).
$h_{def}$	The elastic deformation (m).
$h_o$	The minimum oil film-thickness (m).
H	The non-dimensional film-thickness.
$H_{def}$	The non-dimensional elastic deformation.
$H_o$	The non-dimensional minimum film thickness.
HD	Hydrodynamic lubrication
$K^e$	The element stiffness matrix
$K^G$	The global stiffness matrix
$K_{ij}$	The one-dimensional elastic deformation stiffness
mm	The number of nodes in y-direction.
N01	The boundary condition
ndf	The nodal degree of freedom
nel	The number of elements
nnel	The number of nodes per element
nn	The number of nodes in x-direction.
$N_i$	The shape function
p	The pressure (Pa).

PH	Hertz contact pressure (Pa).
P	The non-dimensional pressure.
R	The equivalent radius of curvature (m).
SSR	Sliding to roll ratio
T	The temperature (°C).
u	The average velocity of the upper and lower surfaces
U*	The velocity parameters.
w	The applied load (N)
W	The non-dimensional load
W*	The load parameters
x, y, z	The coordinates
Xo, Xe	The non-dimensional inlet and outlet coordinates
z	The coefficient of viscosity-pressure formula.

### LIST OF SYMBOLS

Abbreviations	Meaning
$\alpha$	The pressure coefficient of oil viscosity–pressure formula
$\varepsilon$	The Reynolds coefficient.
$\lambda$	The parameter of the coefficients $\varepsilon$ .
$\eta$	The oil viscosity (Pa.s).
$\eta^o$	The oil viscosity at $P_o$ .
$\eta^*$	The non-dimensional viscosity of the lubricant.
$\mu$	The friction coefficient.
$\rho$	The lubricant density ( $\text{Kg/m}^3$ ).
$\rho^*$	The non-dimensional density of lubricant.
$\Delta X$	The non-dimensional increment between the nodes of the mesh.

# CHAPTER ONE

## INTRODUCTION

---

### 1.1 General Review

Friction occurs everywhere in our everyday life, and in many respects, it is a basis force. For instance, friction makes vehicle move and stop, but sometime it is unwanted. Such as, it can cause power losses in machines and decrease the lifetime of contacting components 'due to wear'. So friction must be reduced in these conditions. At recent days, with increasing demands on industry to decrease energy consumption and emissions, the aim to increase the efficiency of machine components is maybe bigger than ever. The most public technique to decrease friction and avoid wear is by using lubrication, for which lubricants is used to detach contacting surfaces. Friction can be minimized to a small portion of the not lubricated state. The efficiency of the machineries will significantly improve, and the lifetime of the engine elements will significantly extend. Generally, there three regimes of lubrication, which defined according to their range of coefficient of friction [1], as shown in Stribeck curve in figure (1.1).

- **Boundary lubrication:** a main part of the load is supported by the direct contact of the surface asperities. This regime is recognized by high friction coefficients.
- **Mixed lubrication:** in this regime both the direct asperities contact, and the film of lubricant are support the load. The friction coefficient for this regime is less than that in the boundary lubrication.
- **Full film lubrication:** the surfaces in contact are completely separated by the lubricant film. The coefficient of frictions are relatively low.



In this study, the main interest is “full film lubrication” regime. When the contacted surfaces are completely separated by the film of lubricant.

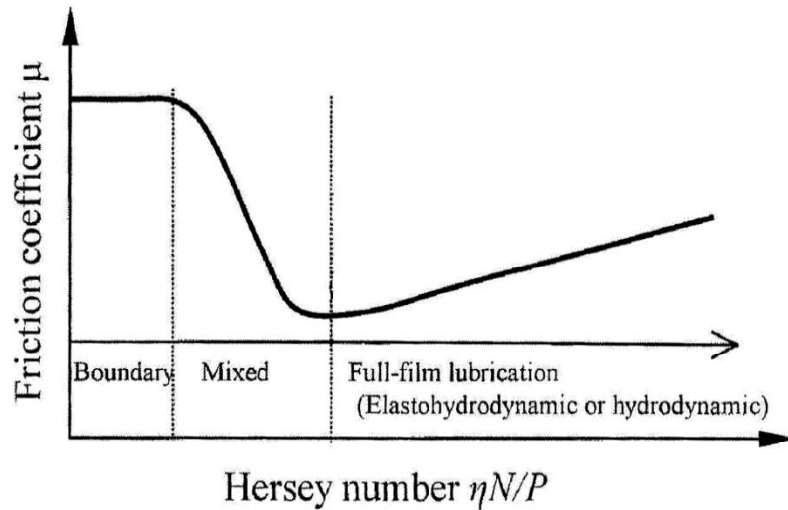


Figure 1.1 Stribeck curve for the lubrication regime [1].

Studying and understanding lubrication behavior, is a high interest for researchers, because it is an efficient technique to decrease power losses and avoiding wear. In addition, the behavior of the lubricants film amongst contacting components is of great significance in determining its performance. In many cases, the shapes of the contacting surfaces determine the film thickness. This case known as “Hydrodynamic lubrication” (HD) [2], which is shown in figure (1.2). Nevertheless, when the pressure is high enough as compared to the stiffness of the contacting surfaces, the elastic-deformation of the surfaces cannot be neglect, and this can heavily influence the film shape of the lubricant. Such lubrication is more complicated state, and it is known as “Elasto-hydrodynamic Lubrication” (EHD or EHL). Where EHL is the phenomena that occur in the lubricated elements having a small contact area and heavily loaded. The pressure generated enclosed the lubricating films are fully high to causes elastic deformation in the contact elements, such as in gear and bearing. Nevertheless, the theory of EHL is not limited only for highly loaded states; it is applicable to all cases where the contact between the film

shape of lubricant and the elastic-deformation of the components in contacts cannot be ignored. Moreover, at high loads, the film shape is typically very thin, the pressure in the contact area is very high and the lubricant passes into the contact in very short time, approximately a hundredth of a second. All of these details make it not easy to assume measurement during physical experiments. With the development of computational ability in the last years, the numerical model has been widely used to study the behavior of EHL contacts [3].

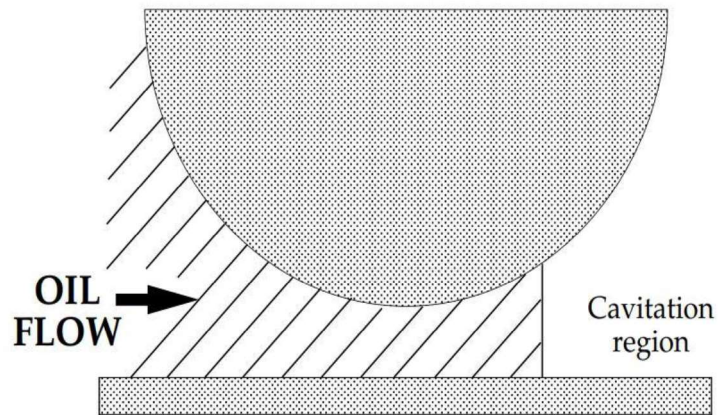


Figure 1.2 Representation of hydrodynamic lubrication [2].

A very interesting feature of the flow is a “steep pressure spike” that occurs in the exit area at highly loaded case. Where it is a physical property that is easily fluctuating, in spite of being very steep. The shape and positions of the spike are both fluctuate with the load condition. The other important future section of EHD is the outlet free boundary or “cavitation’s point”. Where the lubricant’s pressure drop-up to negative pressure, equal to the vapor pressure, and generally it taken to be zero. For instance as compared with HD problems, these details are increased the complication of EHL problems. For a usual EHD contact problems, the required information’s consist of, the pressure and film-thickness profiles, and the cavitation position. Figure (1.3) shows a typical EHL lubrication case. Generally, there are two categories of contact types: “conformal contact” and “non-conformal contact”. When the contact surfaces

meet over a line or at a point preceding to any deformation a non-conformal contact will formed. In this contact, the contact region is very small, compared to the size of the contacting components and loads are highly concentrated in this area. This contact type usually fall into EHL lubrication regime. These cases of EHL contact divided into two types, according to the dimension of the contact: the “line contact” 1D and the “point contact” 2D [1].

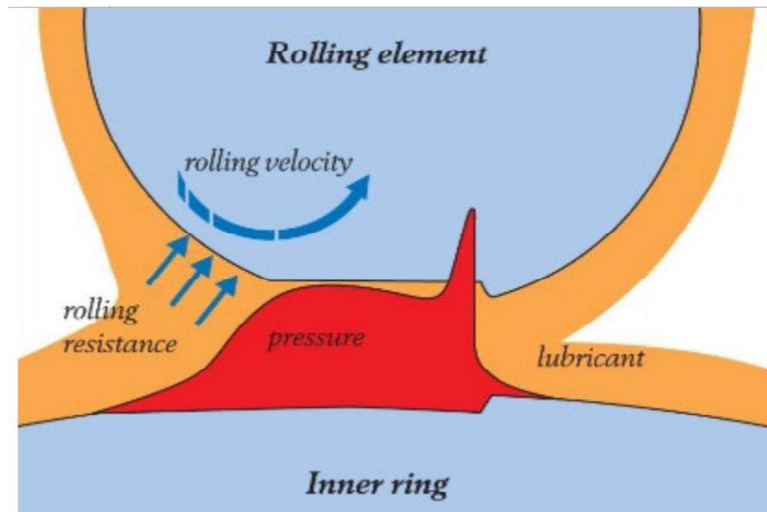


Figure 1.3 Representation of EHL between two deformed surfaces [3].

## 1.2 Cavitation

In any problem of EHL contact, the pressure increases gradually, and becomes very high as compared with the ambient pressure. Then it starts to decrease, at the exit of contact region the pressure become negative (below vapor pressure) this point known as a ‘cavitation point’. Where the cavitation is a phenomenon in which rapid changes of pressure in a liquid lead to the formation of small vapor-filled cavities, in places where the pressure is relatively low. When subjected to higher pressure, these cavities, called "bubbles" or "voids", collapse and can generate an intense shock wave. There are several techniques for handled this phenomenon. A common model for it is the Swift-Steiber boundary condition or the Reynolds exit condition, which sets

the pressure equal to zero at the location  $x_e$  where cavitation starts. Another technique that used for the cavitation is the Gauss-Siedel iterative scheme, and the linear complementarity problem (LCP) approach [4]. Another method, known as the “Penalty Method”, introduced by Oden and Wu [5] also is used to manage the cavitation conditions. A penalty term is added into the Reynolds equation, which impose the negative pressures to be zero in a weak form. At the cavitation position both ( $P = 0$  and  $\partial P/\partial X = 0$ ) can be weakly satisfied. For the penalty method, the grid don’t need to move, but the computational area should be large enough to make sure that the cavitation position is located inside it. The cavitation position can be captured automatically with the penalty method, but the accuracy of it depends on the local mesh size and the order of the local foundation functions.

### **1.3 Convergence and Solution of EHL Problems**

The real solution of EHL problems may not be calculated even if the pressure has been converged. The reason is that unlike the film-thickness of an ordinary “hydrodynamic lubrication” problems is fixed. The given load must modify the film-thickness of an EHL problems, so that the pressure will vary. This denote that the load isn’t obtained but is pre given, and the summation of pressures must be equal to it. So according to the relationship between the film-thickness and loads, it must be known that [2]:

1. When  $\Delta W$  is greater than zero, the summation of pressures-is smaller than the applied load. Hence, it is required to decrease the film thickness that is to reduce  $H_0$ ; in order to increases the pressure.
2. When  $\Delta W$  is less than zero, the summation of pressures is larger than the applied load. Hence, it is required to increase the film thickness that is to increase  $H_0$ , in order to reduce the pressures.

The key problem is that  $H_0$  could dramatically make the pressure distribution change. Thus, the solving process is easily to be divergent, when  $\Delta H_0$  is not little. Nevertheless, when  $\Delta H_0$  is too little, it will rise the computational workload [6].

#### **1.4 Literature Review**

In this research a number of techniques for solving EHL problem was used. The developed program was used to print out the pressure and film thickness profile for line and point contact cases. The experimental work was included the estimation of friction coefficient in EHL point contact problem.

Osborn Reynolds in 1886 established the basics of theory of fluid film lubrication, that famous now as “The Reynolds Equation”, after the experimental study of friction in lubricated journal bearings of Tower (1883). The studies of EHL are dominated by efforts to solve EHL problem of line contacts, combining full solutions of the elastic-deformation.

In the late 1950's onwards, Dowson D and Higginson [7] produce a series of calculated solutions of the EHL line contact problems. They separated the governing equations of EHL and solved it by using an iterative procedure. However, because of the iterative method are failure for very highly loaded problems, the authors applied the inverse method (that introduced by Ertel) to EHL problems of line contacts in 1959s. In addition, calculated solutions through a wide range of velocities, loads, and material properties, and expressed in terms of three dimensionless groups to predict initial minimum film-thickness ( $W^*$ ,  $U^*$ ,  $G$ ). This work approves the main features of EHL contacts, a nearly flat central area of the contact with a contraction at the rear, where the film-thickness equals to (70-80%) of the central amount. The pressure is close to Hertzian at highly loads, on most of the contact area, with a slow build up in

the inlet area, and a pressure spike occurs at the beginning of the contraction at the contact rear [6].

By using an iterative procedure, which established in the 1950s by D. Dowson, Ringer et al. in 1970s [8] presented a full numerical solution of the two-dimensional EHL problems or point contacts. Then Hamrock and D. Dawson develop this to elliptical contact to present equations of minimum and central film-thickness in terms of four dimensionless parameters, involving one describing ellipticity [9-12]. In 1970s, the “finite element method” (FEM) has been applied to EHL problems. However, Taylor and O’Callaghan [13] considered a line contact, and then Rohde and Oh [14] considered point contact. The common direct technique, which is most widely used, is not stable at highly loaded and very thin films case. In addition, the all solutions in use are highly time-consuming; until allowing for the progressive growth in the speed and power of computers that is taking place. This period showed some attempts to tackle this problem by using the newton-Raphson method that introduced by Okamura in the 1982s [15] and solved by Houpert and Hamrock in the 1988s [16].

The importance of numerical study in EHL problems at last years has been on the thin-film, where the film of lubricant became very thin at asperity conjunctions that is, “mixed lubrication”. Zhao J, et al [17] showed particular interest that how and when the film of lubricant was fully break down in these areas. Furthermore, Venner CH, et al [18] applied the fast EHL solvers that established in the late 1990s to a range of transient problems, involving behavior during the startup of motion and Wang QJ and Zhu D [19] work on moving roughness.

Evans and Hughes [20] based on the half space approach, introduce a new numerical method “differential deflection method”. This method has an

advantage, which is to use the data from relatively little points in the area of contact to compute the elastic deformation, at all points. In other words, the effect of pressure acting at a point was decrease to a limited area of that point. So, this method resulted in a less dense matrix compared to the half space method for elastic deformation. Evans, et al [21] applied this technique to a line contacts, then expanded it to a point contacts states [22, 23].

Hongqiang Lu et al [24, 25] in (2006) solve 1D and 2D EHL problems by using a high-order finite element scheme, based on the “Discontinuous Galerrkin” (DG) technique. The author showed that technique is applied successfully to steady state line contact problems. The free boundary is taken accurately, by using the moving grid method and the penalty method respectively. The extension of that technique to 2D state or point contacts is simple. Nevertheless, the computation in the 2D state is more complicated due to extra dimensions. Hence, the author employed P-mulltigrd, to increase the efficiency, and because of the complicated of the free boundary condition in the 2D state, the author used the penalty method to handle the cavitation condition. The author also used the FDM to discretize the equation of problem, and make a Comparison between FEM and FDM about the solution time consuming and the solution accuracy [26].

D. E. Hart, et al [27, 28] applied ad-joint error estimation techniques to EHL problems. The difference numerical techniques are used in this study. Uniform mesh finite-difference approximations, have been used to discretize the equations of problem, with multi-grid used to efficiently solve the equations, and spatial adaptively added through multi-grid patches. The ad-joint problems have been solved, using ‘standard linear algebra packages’. The cavitation condition in this study is captured correctly by using a sliding mesh. The

Newton-Raphson method is used for line contact, and shows the effects of the load and speed change, and mesh size on pressure profile and film thickness.

W. Habchi, et al [29, 30] applied a numerical method to calculate the solution of classical “linear elasticity” equation to find the elastic-deformation. This equation use just the data at the neighboring points to compute the elastic-deformation at a point in the domain. Thus, the resultant matrix was highly sparse and made it simple to get the solution without any special processing for convergence. But this technique has a drawback that is need to solve elasticity equation in a two dimensional case for line contact and a three dimensional case for point contact problems. This can be reduced by using a fine mesh in the area of interest and a coarse mesh elsewhere. The other advantage of this technique is that it yields additional solution information such as displacement, and derived fields such as stress, throughout the solid components, which is not possible using traditional half-space method. However, the relatively high computational cost of this method has so far prevented its wide spread use. The author also presented a new method for solving the fully coupled iso-thermal EHL problem by using a finite element discretization of the corresponding equations. The non-linear system of equations was solved by using a Newton procedure, which provides faster convergence rates. The author used suitable stabilization techniques to extend the solution to the case of highly loaded contacts. The complexity is the same as for classical algorithms, but an improved convergence rate, a reduced size of the problem and a sparse Jacobean matrix are obtained. Thus, the computational effort, time and memory usage are considerably reduced [31].

Sarfraz A., et al [32, 33] used an efficient numerical solution of EHL problems, to get stabilized results for heavy loaded line contact cases, and middling loaded in point contact states. The major contributions of that study



are propose, analyze, and implement a novel optimal preconditioned for the Newton linearization of this algebraic system of equation, and estimate the development of efficient finite element meshes, through both manual setting, and the use of adaptive mesh refinement based on a posteriori error estimation and control. In their study, the authors developed locally adaptive solution scheme for a fully coupled EHL point contact problems. The authors presented the results for line and point contact problems, to validate the implementations, and to allow a comparison of the performance and efficiency of the proposed solution strategies compared to the use of a state of the ‘art sparse direct solver’ at each Newton step. The authors showed ‘these results demonstrate that the preconditioned iterative approach was both computationally and memory superior to the sparse direct solver. Most importantly, both the computational and memory costs were seen to grow linearly with the number of unknowns.

For Experimental part, Spikes HA, et al [34, 35] focused on applying optical interferometry techniques to study fluid film distribution and compare measurements with theoretical estimations. Two new experimental methods have emerged, the first one by Dwyer-Joyce RS, et al [36] was the use of ultrasonic to measure film thickness in EHL contacts. The second by Cann PM and Spikes HA [37] was the use of a pressure-sensitive surface film to obtain high-resolution maps of pressure from EHL contacts.

Zhang Y, et al [38] produce a new improved optical EHL test rig in ball on ring contact for the analyses of film-thickness at high velocity up to (52 m/s). The Results showed that, film-thickness follows classical prediction at low velocity, deviates but still increases at medium velocity and finally decreases at very high velocity. Severe sliding occurs at high velocity, the film-thickness profile changes from flat to wedge along the direction of motion.

Wang WZ, et al [39] produce a numerical technique to simulate sliding friction between engineering surfaces with (3D) roughness in point contacts. The authors used “Universal Material Tester” UMT to measure friction at a fixed load, and different sliding speeds in rotary or reciprocal motions. Their Results showed a general agreement between the simulations and experiments. In addition, the authors found that surface features, such as roughness amplitude and patterns, might have a significant effects on the critical speed of transition from hydrodynamic to mixed lubrication.

Björling Marcus [40] used the “Mini Traction Machin” MTM testing machine in the experimental studies. The author showed how friction varies through a wide range of running conditions when changing parameters like; the viscosity of lubricant, base oil type, surface roughness and temperature of lubricant. Also used these measurements to predict the friction behavior in a real gear application. Björling M, et al [41] made a friction test in a ball on disc test rig. In addition, calculated the friction coefficient with velocity and slide to roll ratio (SRR). The authors made a test with a number of parameters that varied while studying the coefficient of friction, such as; lubricant viscosity, lubricant temperature, and slide to roll ratio. The results of this study showed that the friction behavior can be strongly affected by varying; surface roughness, operating temperature, and base oil viscosity.

Cousseau T, et al [42] used three different lubricating greases and their bleed and base oils, were compared in terms of film-thickness in a ball on disk test rig, through optical interferometry. The authors used Hamrock’s equation [19] in theoretical part to calculate the film thickness; the result showed a good agreement with oil film thickness that measured.

## **1.5 Research Aims**

Although much work has been done in the field of EHL over the years, there are still areas where more research is needed for a better understanding of the EHL problems. The main aims of this study can be summarized as:

- I. To study the phenomenon of EHL, for the line- and point-contact problems.
- II. Investigating a numerical analysis of highly loaded EHL problems, and implementation of different algorithms in the analysis.
- III. Building a computer programs based on the adopted algorithms using finite element method and finite difference method to find pressure distribution and elastic deformation.
- IV. Using an experimental method to investigate how changes in several parameters influence the friction coefficient in EHL using a ranges of oils grade and loads.
- V. Making experimental tests on EHL to measure friction coefficients using number of engine oils as a lubricant.